Real Variance Analysis of Monte Carlo Critical Rod Position Search Calculation in Sodium-cooled Fast Reactor

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***Keywords : Monte Carlo, Critical Rod Position, Real Variance, SFR**

1. Introduction

The reactivity of the advanced reactors, such as sodium-cooled fast reactors (SFRs) is primarily controlled by the individual movement of control rod and, the rodded operation introduces several concerns in the reactor design and safety analysis. Firstly, since the power distributions are distorted by the local insertion of the control rods, the reactivity uncertainty is propagated through the control rod position to the power distributions. Secondly, the depletion of the neutron absorber material such as B-10 in the control rod can lead to swelling of control rod and reduce the shutdown margin. Therefore, it is necessary to predict the critical rod position to optimize the reactor performance and ensure the safety.

Nowadays, the Monte Carlo (MC) reactor analysis codes are widely applied for the advanced reactor designs due to its the high-fidelity simulation capability, which allows for handling complex geometries and nuclear data without the need for complicated nuclear data processing stages for homogenization and group condensation. However, the application of the MC codes to the critical rod position search calculation has been limited due to 1) the lack of the built-in the critical geometry search capability and 2) instability issues arising from nonlinear iterations, such as the secant-like methods and the linear regression.

Recently, the inline critical rod position search method based on the neutron balance approach has been developed and implemented in the McCARD [1],[2], the MC reactor analysis code developed by the Seoul National University. The proposed method has demonstrated stable and accurate prediction performance in the critical rod position search calculation. However, in this method, the critical rod position is determined by the tally results of the previous cycles (i.e., iteration), it may introduce additional inter-cycle correlations in the effective multiplication factor (keff) and the critical rod position. In this context, this paper investigates the real variance of the critical rod position and the keff in the inline critical rod position search calculation using a typical SFR test problem.

2. Methodology

2.1. Inline Critical Rod Position Search Calculation

The detailed descriptions of the inline critical rod position search method based on the neutron balance approach can be found in Ref. [2]. The resulting iterative rod position update equation is given by:

$$x^{(n+1)} = \left(\frac{P^{(n)}/k_{target} - R_{NCR}^{(n)}}{R_{CR}^{(n)}}\right)\left(x^{(n)} + \Delta_x\right) - \Delta_x, \qquad (1)$$

where k_{target} is the user-provided target keff, $x^{(n)}$ is the total insertion length of the control rod banks at the target criticality condition, which corresponds to the critical rod position at the *n*-th fission source iteration, Δ_x is an arbitrary stabilization parameter introduced to avoid the numerical instability at the all-rod-out (ARO) condition, which is set to 10 cm in this study, $R_{CR}^{(n)}$ is the neutron absorption rate by the control rod, $R_{NCR}^{(n)}$ is the neutron loss rate due to the leakage and absorption except by the control rod, and $P^{(n)}$ is the fission neutron production rate.

In the MC criticality calculation, the parameters $R_{CR}^{(n)}$, $R_{NCR}^{(n)}$, and $P^{(n)}$ can be calculated for each iteration based on the moving average of the latest N_{MA} iterations to reduce the stochastic errors in the rod positions, where N_{MA} denotes the moving average length. The critical rod position is determined by Eq. (1) for each iteration during both inactive and active iterations. The keff can be also estimated as usual and utilized to verify whether the k_{target} is attained within a confidence interval.

2.2. Real Variance Analysis

The iterative update of the control rod position may induce the inter-cycle correlation in the rod position itself and also affect the correlation of the keff. Considering that the sample mean of the tally quantity Q and its sample variance are obtained from the N active iterations, respectively, as:

$$\bar{Q} = \frac{1}{N}\sum_{n=1}^{N} Q^{(n)}, \qquad (2)$$

$$\sigma_S^2 = \frac{1}{N(N-1)}\sum_{n=1}^{N}\left(Q^{(n)} - \bar{Q}\right)^2, \qquad (3)$$

where $Q^{(n)}$ is the tally quantity at the n-th active iteration.

From Ref. [3], the real and apparent variances of the sample mean are derived, respectively, as:

$$\begin{aligned}\sigma_R^2 &\equiv E[\bar{Q}^2] - E[\bar{Q}]^2 \\ &= \frac{1}{N}\sigma^2\left[Q^{(n)}\right] + \frac{2}{N^2}\sum_{l=1}^{N-1}(N-l)C_R[l],\end{aligned} \quad (4)$$

$$\begin{aligned}\sigma_A^2 &\equiv E[\sigma_S^2] \\ &= \frac{1}{N}\sigma^2\left[Q^{(n)}\right] - \frac{2}{N^2(N-1)}\sum_{l=1}^{N-1}(N-l)C_R[l],\end{aligned} \quad (5)$$

where $\sigma^2\left[Q^{(n)}\right]$ is the variance of the MC tally Q at a single iteration and $C_R[l]$ is the real lag l covariance expressed as:

$$C_R[l] = \mathrm{cov}\left[Q^{(n)}, Q^{(n+l)}\right] \text{ for } n = 1, \ldots, N-l. \quad (6)$$

From Eqs. (4) and (5), the variance bias can be expressed as:

$$\sigma_A^2 - \sigma_R^2 = -\frac{2}{N(N-1)}\sum_{l=1}^{N-1}(N-l)C_R[l]. \quad (7)$$

Based on the N_b independent MC runs with different random number sequences, the real and apparent variances, and the real lag covariance can be estimated, respectively, as:

$$\sigma_R^2 \cong \frac{1}{N_b - 1}\sum_{i=1}^{N_b}\left(\bar{Q}_i - \bar{\bar{Q}}\right)^2 \quad \text{with} \quad \bar{\bar{Q}} = \frac{1}{N_b}\sum_{i=1}^{N_b}\bar{Q}_i \quad (8)$$

$$\begin{aligned}\sigma_A^2 &\cong \frac{1}{N_b}\sum_{i=1}^{N_b}\sigma_{S,i}^2 \\ &= \frac{1}{N_b}\sum_{i=1}^{N_b}\frac{1}{N(N-1)}\sum_{n=1}^{N}\left(Q_i^{(n)} - \bar{Q}_i\right)^2\end{aligned} \quad (9)$$

$$C_R[l] \cong \frac{1}{N_b}\sum_{i=1}^{N_b}\frac{1}{(N-l)}\sum_{l=1}^{N-l}\left(Q_i^{(n)} - \bar{\bar{Q}}\right)\left(Q_i^{(n+l)} - \bar{\bar{Q}}\right) \quad (10)$$

where $Q_i^{(n)}$ is the tally quantity at the n-th active iteration of the i-th independent MC run, $\bar{Q}_i$ is the sample mean obtained from the i-th MC run, and $\bar{\bar{Q}}$ is the estimation of the true mean.

3. Numerical Results

The same SFR test problem dealt with in Ref. [2] was used for the real variance analysis. Figure 1 shows the configurations of the test problem. The primary control rod (PCR) bank consisting of 6 control rod assemblies is manipulated to achieve the target keff of 1.0. Initially, the rod position (h_{PCR}) is 25.6 cm withdrawn from the bottom of the active core. The MC calculation conditions for the real variance analysis are provided in Table I. In the inline critical rod position search calculation, the choice of the moving average length N_{MA} may affect the real variance of the critical rod position. Therefore, five test cases were considered in this study, as shown in Table II, which include $N_{MA} = 1, 2, 5,$ and $10,$ as well as the k-eigenvalue calculation with the predetermined critical rod position ($h_{PCR} = 41.615$ cm).

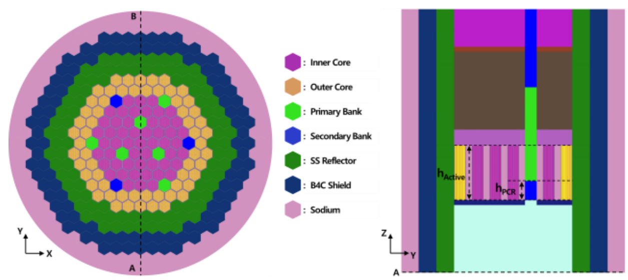

Figure 1. Radial (left) and axial (right) configurations of a typical SFR test problem [2].

Table I. MC calculation conditions for real variance analysis

No. of Independent MC Runs (N_b)	50
No. of Histories per Cycle (M)	100,000
No. of Inactive Cycles ($N_{inactive}$)	40
No. of Active Cycles (N)	100

Table II. Test cases for real variance analysis

Test Cases	Description
NMA_1	Inline rod position update with $N_{MA} = 1$
NMA_2	Inline rod position update with $N_{MA} = 2$
NMA_5	Inline rod position update with $N_{MA} = 5$
NMA_10	Inline rod position update with $N_{MA} = 10$
FIXED	k-eigenvalue calculation with predetermined critical rod position

As the real variance analysis results, the real and apparent standard deviations (STDs) of the keff and the critical rod position (h_{PCR}) are summarized in Tables III and IV, respectively, while Figures 2 and 3 show the lag covariance of the keff and the rod position, respectively, up to a lag length of 10.

The key findings are as follows:

(1) There exists significant negative inter-cycle correlation in the keff when the inline critical rod position search calculation is applied.

(2) The effect of (1) leads to a reduction in real variance of the keff compared to that from the k-eigenvalue calculation.

(3) The effect of (1) also leads to the overestimation of the real variance by the apparent variance for the keff.

(4) The choice of N_{MA} does not affect the real variance of the critical rod position, as the reduced variance in a single-cycle is canceled out by the increased lag covariance.

(5) The ratio of the real to apparent STDs of the critical rod position is roughly proportional to $\sqrt{N_{MA}}$, which can be a good correction factor for the sample STD to estimate the real STD.

Table III. Real and apparent standard deviations of the keff

Test Cases	$\bar{\bar{Q}}$(keff)	$\sigma_R(keff)$	$\sigma_A(keff)$	$\frac{\sigma_R}{\sigma_A}(keff)$
NMA_1	1.00000	0.00006	0.00021	0.29
NMA_2	1.00000	0.00006	0.00019	0.32
NMA_5	0.99999	0.00007	0.00017	0.41
NMA_10	0.99998	0.00009	0.00017	0.53
FIXED	0.99997	0.00019	0.00016	1.19

Table IV. Real and apparent standard deviation of the critical rod position (h_{PCR} in unit of cm)

Test Cases	$\bar{\bar{Q}}(h_{PCR})$	$\sigma_R(h_{PCR})$	$\sigma_A(h_{PCR})$	$\frac{\sigma_R}{\sigma_A}(h_{PCR})$
NMA_1	41.606	0.121	0.085	1.428
NMA_2	41.628	0.097	0.064	1.515
NMA_5	41.638	0.104	0.041	2.504
NMA_10	41.634	0.105	0.030	3.468
FIXED	41.615	-	-	-

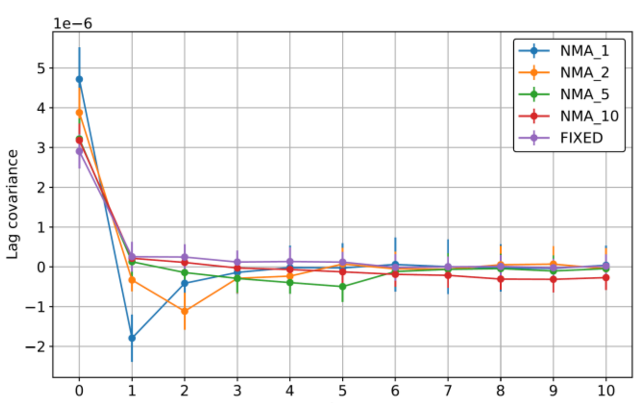

Figure 2. Lag covariance of keff with error bar.

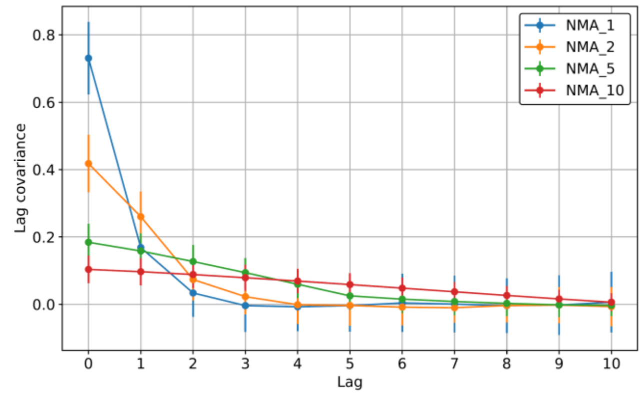

Figure 3. Lag covariance of critical rod position with error bar.

Among the key findings, item (5) can be inferred by introducing the effective sample number $N_{\text{eff}} = N/N_{MA}$ to estimate the sample variance of the mean, considering the moving average length as a factor that reduces the degrees of freedoms (DOFs) in data. Equation (11) shows the modification of Eq. (3) accounting for the reduced DOFs, and Figure 4 demonstrates the feasibility of using $\sqrt{N_{MA}}$ as a correction factor to estimate the real variance.

$$\tilde{\sigma}_S^2 = \frac{1}{N_{\text{eff}}}\left(\frac{1}{N-1}\sum_{n=1}^{N}\left(Q^{(n)} - \bar{Q}\right)^2\right) = N_{MA}\sigma_S^2. \qquad (11)$$

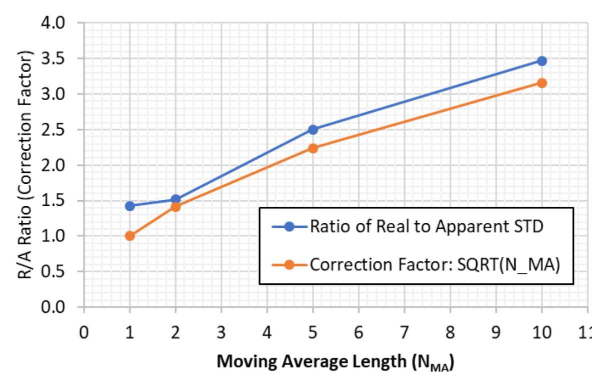

Figure 4. Ratio of real to apparent standard deviations (STDs) and proposed correction factor $\sqrt{N_{MA}}$.

4. Summary and Conclusion

In this paper, the real variance analysis of the MC critical rod position search calculation based on the neutron balance approach was performed. The key findings from the numerical analysis using the SFR test problem provide a useful basis for the understanding the variance estimates from the inline critical rod position search calculation.

Acknowledgement

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT). (No. NRF-2020M2D4A1067573).

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