Transactions of the Korean Nuclear Society Autumn Meeting
Changwon, Korea, October 24-25, 2024

Comparative Study of Dynamic Mode Decomposition Algorithms Applicability on Thermal Hydraulic System Analysis

Ha Neul Na [a*] , Youngsuk Bang [a], and Jaeseok Heo [b]
[a] *Future and Challenge Tech., Gyeonggi-do, Yongin-si, Giheung-gu, Yeongdeok-dong, Heungdeok1ro, 13*
[b] *Korea Atomic Energy Research Institute, Deogjin-dong, Yuseong-gu, Daejeon*
**Corresponding author: nhan@fnctech.com*

***Keywords : Reduced Order Modeling, Dynamic Mode Decomposition, Thermal hydraulics**

1. Introduction

Thermal-hydraulic systems are crucial in many engineering applications, involving complex interactions between heat transfer and fluid dynamics. Simulating these systems accurately can be computationally expensive due to the complexity and size of the models required. Reduced Order Modeling (ROM) is a valuable tool in this context, as they simplify simulations while preserving essential system behaviors. Dynamic Mode Decomposition (DMD) is a technique for analyzing complex dynamical systems by decomposing high-dimensional data into spatial modes and their corresponding temporal dynamics [1].

Surrogate modeling techniques are essential for thermal-hydraulic analyses due to the high computational costs of original systems. The nonlinearity and discontinuous behavior of thermal-hydraulic systems make modeling challenging. Therefore, simplifying these complex systems with surrogate modeling techniques is necessary to enable faster simulations while maintaining critical dynamics.

In this study, DMD-based ROM approach has been applied to thermal-hydraulic systems with the existing engineering thermal hydraulic system code for nuclear power plant accident simulations, i.e. SPACE [2]. A comparative analysis between standard DMD and Dynamic Mode Decomposition with Control (DMDc) has been presented. While DMD focuses on the decomposition of observed data into spatial modes and temporal dynamics, DMDc extends this framework by incorporating control inputs, which are essential for systems where external controls significantly influence behavior. It shows that DMDc could capture the nonlinearity due to control appropriately.

2. Overview of Dynamic Mode Decomposition

The DMD method decomposes snapshots of system's state into a set of modes that capture dominant dynamic patterns [3]. The DMD is performed as follows:

Algorithm 1: Dynamic Mode Decomposition Procedure (DMD)

For Linear System $X' = AX$ (where X contains the system states at time t and X' contains the system states at time $t+\Delta t$).

1) Perform SVD on X to obtain $X = U\Sigma V^*$.
2) Obtain A where $A = X'X^T = X'V\Sigma^{-1}U^T$, and compute A which is the r x r projection of the full matrix A onto POD mode;
   $A = U^T A U = U^T X' V \Sigma^{-1}$
3) Compute eigendecomposition of A , $AW = W\Lambda$.
4) Compute DMD mode Φ, $\Phi = X'V\Sigma^{-1}W$.
5) Reconstruct X $x(t) \approx \sum_{k=1}^{r} \phi_k e^{\varpi_k t} b_k = \Phi e^{\Omega t} b$
   where, $b = \Phi^T x_1$

DMDc extends DMD by incorporating control inputs into the decomposition process [4]. The DMDc procedure involves the following steps:

Algorithm 2: Dynamic Mode Decomposition with Control Procedure (DMDc)

1) Find the dynamic properties of A and B
   $X' = AX + B\Upsilon$
2) Construct the input data matrix
   $\Omega = \begin{bmatrix} X \\ \Upsilon \end{bmatrix}$
3) Find the truncated SVD of input matrix Ω
   $\Omega \approx U\tilde{\Sigma}V^T = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \tilde{\Sigma}V^T$
4) Find the truncated SVD of output matrix X'
   $X' \approx U\hat{\Sigma}V^T$
5) Compute reduced-order approximation of A
   $A = U^T X' V \tilde{\Sigma}^{-1} U_1^T U$
   $B = U^T X' V \tilde{\Sigma}^{-1} U_2^T$
6) Predict X

$$\tilde{x}_{k+1} = A\tilde{x}_K + B\Upsilon_k$$
$$U\tilde{x}_{k+1} = UAU^T U\tilde{x}_k + UB\Upsilon_k$$
$$x_{k+1} = UAU^T x_k + UB\Upsilon_k$$

3. Numerical Demonstration

For comparison of algorithms, the Safety and Performance Analysis Code (SPACE) [2], utilized as the original model, simulated the MIT Pressurizer Test [5]. The surrogates were constructed via DMD and DMDc algorithms and the results were compared and examined.

3.1 MIT Pressurizer Test Modeling

The objective of MIT Pressurizer Test is to investigate heat transfer process occurring in pressurizer. This test involves injecting subcooled liquid into a pressurizer partially filled with saturated water, with injection rates varying between 0.41 and 0.28 kg/s and ceasing after 40 seconds [5]. A balance between interfacial and wall condensation and steam compression determines pressure in test section. The schematics of MIT Pressure Test is shown in **Figure 1** and SPACE model nodalization for this test is presented in **Figure 2**. The pressure within the test section is regulated by the equilibrium between steam condensation and compression, as shown in the pressure distribution presented in **Figure 3**.

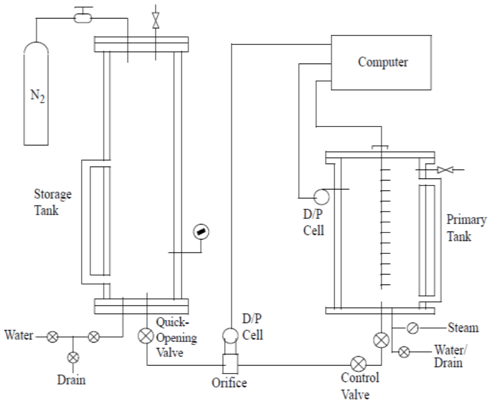

Figure 1. Schematics of MIT Pressurizer Test Facility

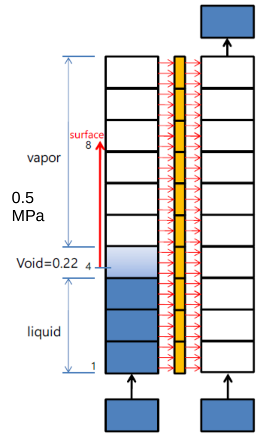

Figure 2. SPACE Nodalization of MIT Pressurizer Test

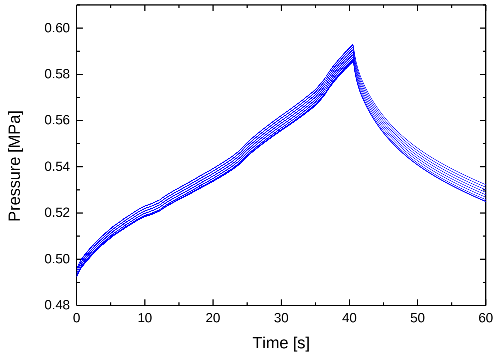

Figure 3. Pressure Distribution of MIT Pressurizer Test

3.2 DMD Algorithm Application

DMD algorithm was applied to model the dynamic behavior of pressure distribution in MIT pressurizer. The snapshot matrix was constructed by stacking the pressure values of SPACE nodes and injection flow rate, i.e., 11 variables. Firstly, the reduce basis was examined by investigating the singular values of the snapshot matrix. As shown in **Figure 4**, the singular values were decreased very rapidly, which implies that only 2 basis vectors would capture the most of the pressure variations. In subsequent analysis, only two basis vectors are used for reduced order DMD estimate, i.e., truncation SVD in Step 2 of Algorithm1 and Step 4 of Algorithm 2.

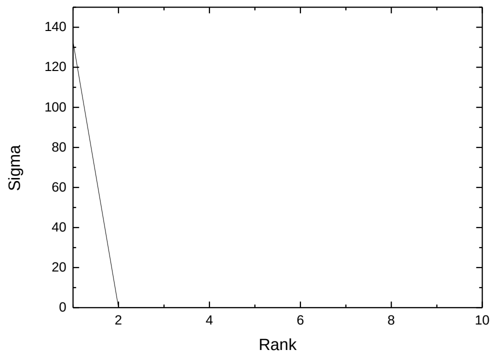

Figure 4. Singular Value Spectrum of Snapshot Matrix

The results of applying DMD to the system are presented in **Figure 5**. **Figure 5** demonstrates how the DMD-based ROM effectively captures the dominant modes and temporal dynamics of the system, thereby providing a reduced-order representation that preserves key system behaviors while significantly reducing computational complexity.

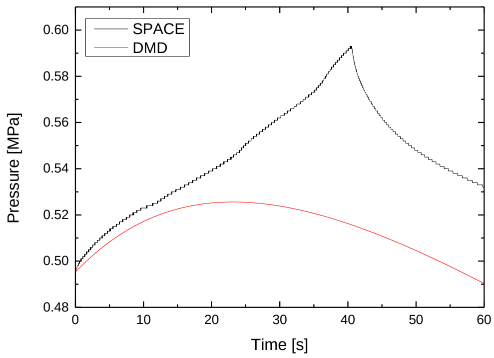

Figure 5. SPACE Calculation vs. DMD Reconstruction

3.3 DMDc Algorithm Application

Following the analysis and application of Dynamic Mode Decomposition (DMD), Dynamic Mode Decomposition with Control (DMDc) was applied to the same system. In the DMDc approach, liquid injection speed was used as the control input. **Figure 6** shows the results of applying DMDc to the system with liquid injection speed as the control parameter. It is important to note that the DMDc predict the system behavior much more accurately and the discontinuous change was captured appropriately.

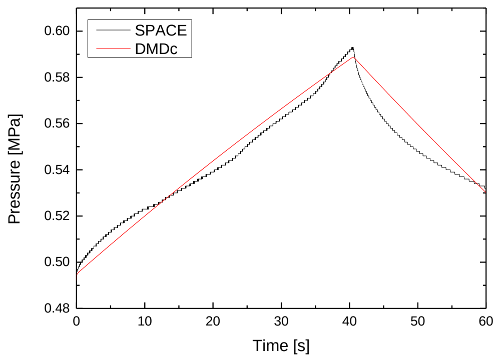

Figure 6. SPACE Calculation vs. DMDc Reconstruction with Control Input (Insurge speed)

To further investigate the impact of control parameters, DMDc was performed again with vapor temperature and vapor enthalpy as the new control inputs which are variables of continuity, momentum and energy equations. The results of this analysis are shown in Figure 7. This new set of control parameters allows for a comparison of how different control inputs influence the system's behavior and the accuracy of the predictions.

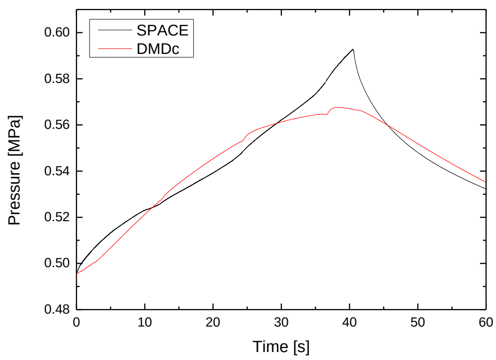

Figure 7. SPACE Calculation vs. DMDc Reconstruction with Control Inputs (Temperature and Enthalpy)

4. Conclusion

In this study, both Dynamic Mode Decomposition (DMD) and Dynamic Mode Decomposition with Control (DMDc) were applied to analyze and model the dynamics of a thermal-hydraulic system. The results reveal that while the DMD approach provided some insights into the system's behavior, it was insufficient in capturing the full dynamics of the system due to high nonlinearity. DMD was unsuitable to accurately model the nonlinearities of system.

In contrast, the DMDc method, which incorporates the effect of control inputs—in this study, the liquid injection speed—demonstrated significantly improved performance. The results showed that DMDc could predict the critical dynamics and addressed effectively the nonlinearities that comes from the control inputs. This improvement emphasizes the importance of identifying control inputs in the reduced order modeling of thermal-hydraulic systems, especially where operator action or control actuation significantly influence system behaviors.

To further evaluate the influence of control parameters, additional DMDc analyses were performed using different control variables, such as vapor temperature and vapor enthalpy. These analyses confirmed that the results varied significantly with different control inputs. This observation emphasizes that it is important to select the appropriate control parameters in DMDc.

Future work will focus on applying DMDc to more complex experimental setups and systems with multiple interacting control inputs, e.g., accident analysis.

ACKNOWLEDGMENT

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (Grant No. RS-2022-00144466).

REFERENCES

[1] J. Nathan Kutz, Steven L. Brunton, Bingni W. Brunton, and Joshua L. Proctor, Chapter 1 : Dynamic Mode Decomposition: An Introduction, Society for Industrial and Applied Mathematics, 2016.
[2] Sang Jun Ha, Chan Eok Park, Kyung Doo Kim, and Chang Hwan Ban, “Development of the SPACE Code for Nuclear Power Plants,” Nuclear Engineering and Technology, 43, 1, 2011.
[3] Peter J. Schmid, Dynamic mode decomposition of numerical and experimental data, J. Fluid Mech., vol. 656, pp. 5-28, 2010.
[4] J. L Proctor, S. L. Brunton and J. N. Kurz, Dynamic mode decomposition with control, Society for Industrial and Applied Mathematics, 2014.
[5] H. R. Saedi, Insurge Pressure Response and Heat Transfer for PWR Pressurizer, Massachusetts Institute of Technology, 1982.