Weighting Function for Cross Section of Node with Repetitive Control Rod Insertion

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1. Introduction

In the analysis of pressurized water reactor (PWR) cores, neutronic calculations often use two-group neutron cross sections based on depletion models that assume the control rods are not inserted. This approach is typically valid for reactors operating with all control rods withdrawn throughout the operational period, relying on soluble boron as the primary means of reactivity control. However, this method may not be suitable for cores that do not use soluble boron, where control rods are actively used to manage reactivity over both short-term and long-term periods, including during fuel depletion. In regions of the core where control rods are inserted, the neutron spectrum is significantly hardened compared to regions without inserted rods. This difference results in considerable differences in the neutron cross sections of each isotope within these regions. To address this, Park et al. [1] propose a methodology involving the preparation of two distinct sets of cross sections—one for conditions with control rods withdrawn and another for conditions with control rods inserted. They advocate using a history index, defined as the ratio of integrated burnup with control rods inserted to total burnup, to weight the cross sections. This approach has demonstrated the potential to reduce reactivity differences from approximately 3000 pcm to about 1200 pcm in simplified control rod insertion scenarios.

This study extends the work of Park et al. by considering various repetitive control rod insertion states to evaluate reactivity differences more comprehensively. Additionally, a modified history index is proposed to further minimize these differences. The DeCART2D [2] code, developed by the Korea Atomic Energy Research Institute, is employed for depletion calculations of a typical fuel assembly designed for operation without boron, using $UO_2-Gd_2O_3$ and $B_4C-Al_2O_3$ as burnable absorber materials.

2. Test of History Index Method

2.1 Cross Section Weighting by History Index

Two sets of cross sections are prepared: one corresponding to the condition with the control rods out and the other with the control rods in. These sets are used to calculate the cross section of a node, taking into account repetitive control rod insertions. The cross section of a given isotope is determined by combining these two sets, weighted by a history index [1], as follows:

$$
\sigma = \omega \sigma_{RI} + (1 - \omega) \sigma_{RO},
$$

where,

$$
\omega = \frac{1}{B} \sum_{i} (B_{li} - B_{0i}),
$$

 σ_{RI} and σ_{RO} are cross sections based on the control rod in and out, respectively, B_{Ii} and B_{Oi} are *i*-th control rod in and out burnups and B is the total burnup.

2.2 Test Fuel Assembly Configuration

A fuel assembly used for a boron free operated core is selected for the test of the cross section weighting method. The fuel assembly has typical $17x17$ UO₂ array at which $4 \text{ UO}_2\text{-Gd}_2\text{O}_3$ rods and $24 \text{ B}_4\text{C}-\text{Al}_2\text{O}_3$ burnable absorber rods as shown in Figure 1.[3]

Fig. 1. Test Fuel Assembly Configuration

2.3 Test Cases and Results

Two cases are selected to assess the impact of history index weighting on cross sections for the rod-out and rod-in conditions. Case 1 involves a fuel assembly with control rods inserted from 20 to 60 MWD/kgU. Case 2

involves control rods inserted from 20 to 30 MWD/kgU and from 40 to 50 MWD/kgU. Figures 2 and 3 present the reactivity differences calculated using rod-out and rod-in cross sections, as well as the weighted cross sections for these two test cases. The reference reactivity is determined based on depletion calculations for both rod in and out.

Fig. 2. Reactivity Difference of Test Case 1

Fig. 3. Reactivity Difference of Test Case 2

3. Cross Section Weighting Function

3.1 Best Estimated Weighting Factors

The best-estimated weighting factors for the two test cases described in the previous section are determined iteratively, ensuring that the maximum reactivity difference is less than 25 pcm. Figures 4 and 5 display the optimized weighting factors in comparison to those derived from the history index. These comparisons suggest that weighting the cross sections using the history index—based on a linear summation of the control rod insertion burnup ratio—is insufficient for accurately estimating the node cross sections in scenarios involving repetitive control rod insertions.

Fig. 4. Weighting Factor Comparison of Test Case 1

Fig. 5. Weighting Factor Comparison of Test Case 2

3.2 Optimized Weighting Function

The optimized weighting function is determined using the best-estimated weighting factors obtained iteratively for the various cases listed in Table 1. These cases encompass all possible scenarios for repetitive control rod insertions over a burnup range from 0 to 60 MWD/kgU, with a specific focus on the burnup interval of 10 MWD/kgU. The optimized weighting function is given

$$
\omega=\sum_i w_i,
$$

where w_i is the weighting factor of i -th repetitive control rod insertion that calculated by

$$
t \le T_{RO}
$$

$$
w_i = \frac{t - T_{RI,i}}{t - \alpha_i},
$$

$$
\alpha_i = 0.563 T_{RI,i} + 0.00398 T_{RI,i}^2,
$$

$$
if\ t > T_{RO}
$$

≤

$$
w_i = \frac{w_i(T_{RO,i})}{1 + \beta_i(t - T_{RO,i-1})'}
$$

$$
\beta_i = 0.44 + \frac{1.38}{T_{RO,i} - T_{RI,i}}
$$

Using the above weighting function, the reactivity differences of all cases described in Table 1 are shown in Figure 6, comparing with the results using the history index weighting.

Table 1. Repetitive Control Rod Insertion Case

Fig. 6. Reactivity Differences by Optimized Weighting

4. Conclusions

For nodes with repetitive control rod insertion history, we propose an optimized weighting function derived from the best-estimated weighting factors across various cases. This function accounts for the complexities associated with varying control rod insertion patterns. The application of this optimized weighting function results in cross sections that produce a maximum nodal reactivity difference of less than 500 pcm, demonstrating a significant improvement in accuracy compared to previous methods.

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