Few-Group Transport Analysis of the Core-Reflector Problem in Fast Reactor Cores via Equivalent Group Condensation and Local/Global Iteration

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1. Introduction

In deterministic neutron transport methods, a process called fine-group to few-group condensation is used to reduce the computational burden. However, recent results on the core-reflector problem in fast reactor cores show that use of a small number of energy groups has limitation to describe neutron flux around corereflector interface[1, 2]. Therefore, researches are still ongoing to overcome this limitation.

Recently, the authors proposed [3, 4] i) direct application of equivalently condensed angle-dependent total cross section to discrete ordinates method to overcome the limitation of conventional multi-group approximations, and ii) local/global iteration framework in which fine-group discrete ordinates calculation is used in local problems while few-group transport calculation is used in the global problem iteratively.

In this paper, an analysis of the core-reflector problem is performed in few-group structure using equivalent angle-dependent total cross section with local/global iteration. Numerical results are obtained under S_{12} discrete ordinates-like transport method with scattering cross section up to P_1 Legendre expansion.

2. Methods and Results

2.1 Group Condensation

To get fine-group to few-group condensed cross sections, the fine-group discrete ordinates equation is considered first:

$$\mu_{n} \frac{\partial \psi_{n,g}(x)}{\partial x} + \sigma_{g}(x)\psi_{n,g}(x) = \sum_{l=0}^{\infty} (2l+1)P_{l}(\mu_{n})\sum_{g'} \sigma_{lgg'}(x)\phi_{lg'}(x) + \frac{\chi_{g}}{k_{eff}}\sum_{g'} \upsilon \sigma_{fg'}(x)\phi_{g'}(x), \quad (1)$$

where n indicates discrete angle index, g fine-group energy structure.

If Eq.(1) is summed over the energy within few-group structure ($g \in G$), then few-group discrete ordinates equation is obtained:

$$\mu_{n} \frac{\partial \psi_{n,G}(x)}{\partial x} + \hat{\sigma}_{n,G}(x)\psi_{n,G}(x) = \sum_{l=0}^{\infty} (2l+1)P_{l}(\mu_{n})\sum_{G'} \sigma_{lGG'}(x)\phi_{lG'}(x) + \frac{\chi_{G}}{k_{eff}}\sum_{G'} \upsilon \sigma_{fG'}(x)\phi_{G'}(x), \qquad (2)$$

where

$$\hat{\sigma}_{n,G}(x) = \frac{\sum_{g \in G} \sigma_g(x) \psi_{n,g}(x)}{\psi_{n,G}(x)}, \quad \sigma_{IGG'}(x) = \frac{\sum_{g' \in G'} \sum_{g \in G} \sigma_{Igg'}(x) \phi_{Ig'}(x)}{\phi_{IG'}(x)},$$

$$\upsilon \sigma_{IG}(x) = \frac{\sum_{g \in G} \upsilon \sigma_{Ig}(x) \phi_g(x)}{\phi_{G}(x)}.$$
(3)

As shown in Eq.(3), the group condensed total cross section $\hat{\sigma}_{n,G}(x)$ has angle dependency. Instead of the traditional consistent P_N approximation and extended transport approximation, the authors proposed direct application of the angle-dependent total cross section in the sweeping calculation without any approximation [3,4]. This can be done by giving different total cross section to each discrete angle. This has been programmed in a code TRUST and verified [5].

2.2 Local/Global Iteration

Fig.1 shows a flow chart of the local/global iteration framework used in this study. It is noted that the fewgroup parameters are generated for each fine-mesh because nodal methods in discrete ordinates transport are not so effective and sweeping procedure is effective enough to analyze global problem.



Fig.1 Flow chart of local/global iteration

One of weaknesses of the local/global iteration is the albedo boundary condition updating after global calculation. For the next local calculation, a fine-group albedo boundary condition is consequently assumed to be constant within the few-group structure. However, if overlapped subregion is used for local calculation, solution of the local problem can be used to "modulate" the few-group albedo boundary condition [6].

The equation for albedo modulation is expressed as:

$$\alpha_{g,s}^{modulated} = \frac{\alpha_{g,s}^{local}}{\alpha_{G,s}^{local}} \alpha_{G,s}^{global}$$
(4)

where s denotes local problem center surface, g finegroup index, and G few-group index.

2.3 Numerical Results

It is reported [2] that the use of a small number of few groups in existing methods for the core-reflector problem in fast reactors induces large discrepancy with reference solution.

In this study, a one-dimensional core-reflector slab problem representing fuel and reflector assemblies in a typical fast reactor is considered. The problem configuration is shown in Fig.2 and Table 1.



Fig.2 Problem configuration

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Region	Isotope	Density [atoms/barn-cm]		
Core	Pu239	1.5×10 ⁻³		
	U238	5.0×10 ⁻³		
	Fe56	7.0×10 ⁻³		
	Cr52	1.5×10^{-3}		
	Na23	1.0×10^{-2}		
	O16	1.5×10^{-2}		
Reflector	Fe56	5.0×10 ⁻²		
	Cr52	1.5×10^{-2}		
	Na23	5.0×10 ⁻³		

150-group cross sections are used as a fine-group structure with scattering cross section up to P_1 Legendre expansion in this problem. This 150-group cross section set is obtained by the TRANSX code [7] processing based on JEFF 3.1 library. In group condensation, few-group cross sections are generated in three ways: i) 25-group, ii) 4-group, and iii) 1-group condensation, with angle-dependent total cross sections

Half-overlapped subregions are used for local calculation in this study. The diamond difference scheme with 0.5cm fine-mesh size and S_{12} discrete ordinates are used for both local and global problems.

Table 2 shows k_{eff} values up to 2nd local/global iteration.

Table 2: k_{aff} and its relative errors

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		$k_{e\!f\!f}$	Relative Error [pcm]	
Reference [*] (150-group)		1.389950		
150-group to 25-group	0 th	1.392059	151.8	
	1 st	1.389955	0.4	
	2 nd	1.38995	0.0	
150-group to 4-group	0 th	1.384667	-380.0	
	1 st	1.389854	-6.8	
	2 nd	1.389951	0.1	
150-group to 1-group	0 th	1.365306	-1773	
	1 st	1.389895	-3.9	
	2 nd	1.389953	0.3	

* reference calculation performed in fine-group and verified by comparing with that of the ONEDANT code [8]

In the case of 0^{th} local/global iteration, relative error in k_{eff} is quite large due to initial guess of the albedo boundary condition for local problems.

However, at 1st and 2nd local/global iterations, the k_{eff} results are dramatically improved for all three cases. The reasons are that albedo modulation gives more realistic boundary conditions for local problems and few-group angle-dependent total cross sections are generated equivalently.

The results of scalar flux distribution has similar tendency with that of k_{eff} . The maximum relative error in scalar flux distribution is around 5~10% at 0th local/global iteration. However, that is decreased to a negligible level of 0.01~0.05% at 1st local/global iteration.

3. Conclusions

The spectrum transient effects in the core-reflector problem in fast reactor cores are analyzed by the methodology of equivalent angle-dependent total cross section with local/global iteration. The numerical results show excellent agreement for k_{eff} and flux distribution estimation even in the 150-group to 1-group condensation case.

This encouraging result sheds a light on resolution of an outstanding problem in fast reactor design.

Application to multi-dimensional problems will be studied as a further study.

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