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Verification of FAST Code Using Method of Manufactured Solution

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1. Introduction

FAST code[1] is developed to calculate the DNBR value in the reactor core of SMART. The code involves the discretization of a set of differential equations such as continuum, energy, axial and lateral momentum equation. There is a truncation error which is the difference between the algebraic set of difference equations with numerical approximations and that of the original differential equations. This truncation error degrades the accuracy of a calculation solution. As a result of this error, numerical results describe more diffusion than true solution.

It is possible to quantify the error using the observed order of accuracy and manufactured solution method[2]. Observed order of accuracy(OOA) is to investigate the natural order of accuracy of the code. The natural order of accuracy is also called to the formal order of accuracy(FOA) that is evaluated by the truncation error analysis using the Taylor series expansion of the solution variables The method of manufactured solution(MMS) is a general procedure that can be used to construct analytical solutions to PDEs. MMS is to generate the source term that contended the boundary condition using the manufactured solution. Code verification can be performed to investigate the difference between calculation results and MMS with generated boundary condition. Resultantly, accuracy of code is evaluated through calculated OOA from code.

MMS for single subchannel with 2 lateral flow was derived to estimate the accuracy of FAST code. The derived MMS and OOA was applied to verify the FAST code.

2. Methods and Results

2.1 Evaluation of Order of Accuracy of FAST code

The evaluation of the OOA is a rigorous test, which determines whether or not the discretization error is reduced at the expected rate. By calculating the OOA, one can verify if the code converges to the correct solution or not. If the OOA matches or nearly matches the FOA, the code is able to reproduce the FOA of the numerical method. FOA is determined by evaluating the truncation error using Taylor series expansion of the solution variables.

FOA of FAST code is derived to apply truncation error analysis to governing equations. For a single sub-channel, the energy equation is written as:

$$m_i \frac{\partial h_i}{\partial x} = q_i' - \left(\frac{h_i - h_j}{N_H}\right) w_{ij}' + \left(h_i - \left(\frac{h_i + h_j}{2} + \frac{(h_i - h_j) n_{ij}}{2N_H}\right)\right) w_{ij} \quad (1)$$

To simplify this equation, properties are assumed constant. Equation (1) becomes :

$\frac{\partial h_i}{\partial x} = S_i$ where S is the source term.

After using backward difference scheme, Taylor series expansion is applied for h_i and h_{i-1}. We obtain the final difference equation and truncation error term :

$$\frac{\partial h_i}{\partial x} = \frac{h_i - h_{i-1}}{\Delta x} - \frac{1}{2}\frac{\partial^2 h}{\partial x^2}\Delta x + O\left(\Delta x^2\right) = S_i \quad (2)$$

According to eq. (2), FOA of the scheme is first order because the truncation error term contains $O(\Delta x)$. OOA is directly calculated from simulation results based on the different grid sizes. For a given coarse grid(q_1) and fine grid(q_2), accuracy with grid size is estimated as difference between analytical solution and numerical solution with grid size following as:

$$Err_1(x) = \left|\phi_a(x) - \phi_n(s:q_1)\right| = |a| q_1^2 ,$$

and

$$Err_2(x) = \left|\phi_a(x) - \phi_n(s:q_1)\right| = |a| q_2^2 .$$

Then the OOA can be written as:

$$r = \frac{\ln\left(Err_2(x)/Err_1(x)\right)}{\ln\left(q_2/q_1\right)}$$

2.2 Manufactured Solution for verifying FAST code

In order to calculate OOA, analytical solution is required. An analytic solution of FAST code can be obtained by MMS. To generate an analytical solution, MMS has been applied to the governing equations, In this case, the manufactured solution for enthalpy and axial velocity is chosen as[2]

$$h(x) = h_0 e^{\alpha z} \quad (3)$$

$$U(x) = U_0 + b\sin(\omega x) \tag{4}$$

The analytical form of the transversal velocity should satisfy the mass conservation equation. The analytical form of transversal velocity is given by:

$$W_{IJ}(x) = \frac{Ab\omega}{3\Delta x}\cos(\omega x) \tag{5}$$

Substitution of the chosen manufactured solutions into the energy equation allows the analytical form of the source form Q:

$$Q(x) = \rho A\alpha h_0 \left(U_0 + b\sin(\omega x)\right)e^{\alpha x} \tag{6}$$

The analytical form on the pressure is also obtained by the chosen manufactured solutions:

$$\begin{aligned} P(x) = &-2\rho b\left(U_0 + \frac{b}{2}\sin(\omega x)\right)\sin(\omega x) - \left(\frac{g\beta}{\alpha C_P}\right)h(x) \\ &-\frac{\rho A}{2}\int e^{\lambda z}U^2(x)dx + \rho\beta\omega\int U(x)\cos(\omega x)dx + P_0 \end{aligned} \tag{7}$$

The obtained manufactured solutions and derived source term is used to estimate the numerical discretization errors.

2.3 Results of verification of FAST code

According to three different axial node size, OOA was calculated and compared with analytical solution derived by manufactured solution. The observed order of accuracy as a function of the element size q are presented in Fig. 1. It is noted that the OOA calculated by FAST code agree perfectly with the FOA by truncation error analysis except pressure field. Pressure accuracy is slightly less than order of one.

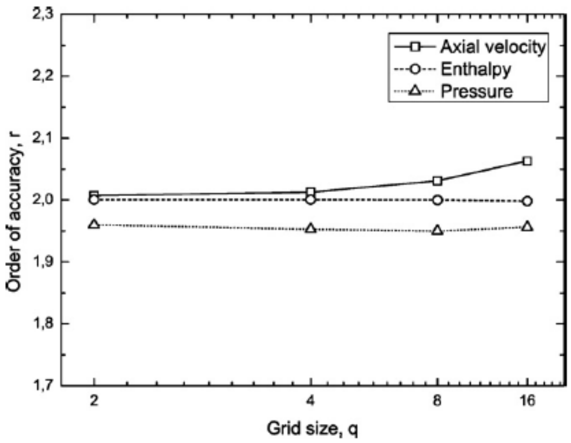

Fig. 1. Observed order of accuracy for axial velocity , enthalpy and pressure with grid size,q.

The enthalpy profile compared to the analytical solution for different grid size are presented in Fig. 2. It is noted that the analytical and numerical solution agree for different grid resolutions. This is explained by the fact that even for coarser grid mesh the relative error for enthalpy is less than 0.3 %. This error is systematically reduced when the grid size increases.

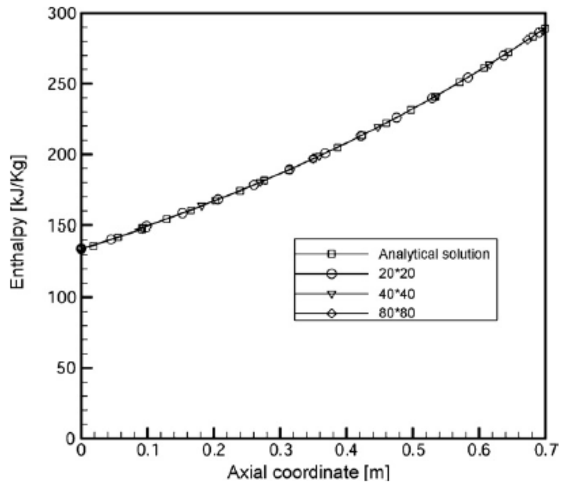

Fig. 2. Enthalpy profile for different grid size and compared to the analytical solution.

3. Conclusions

Verification of FAST code was performed using the method of manufactured solution and observed order of accuracy. Accuracy and consistency of difference equation was verified through these methods.

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