Reviews on the Stability of Numerical Solvers for Multi-fluid Models

Sang Yong Lee*, Chan Eok Park, Eun Kee Kim
KEPCO E&C Company, Inc., 150 Deokjin-dong, Yuseong-gu, Daejeon, 305-353
**Corresponding author:sanglee@kepco-enc.com*

1. Introduction

Multi-fluid models use averaging procedures [1,2] to simplify the complexity inherent in the combined balance equation sets that are separated by the interface and/or structural boundaries. These simplification procedures change the important mathematical properties of balance equations. An annoying mathematical fact about the multi-fluid model is that sometimes it has complex characteristics (CC) [4,5,6], even though the individual constituents have real characteristics (RC). This means that it is not properly posed for the initial value problem [3]. Since this fact was found at early seventies, many researchers have studied to overcome the difficulties [4,5,6]. Some differential terms such as surface tension or diffusion are effective to get rid of CC problem [7]. Interfacial drag term is also helpful to get away with the problem [4].

In this paper, brief reviews on the method taken in two-fluid code RELAP5 [8] will be done and the impacts on having one more field like in SPACE [9] will be investigated.

2. Mathematical Properties for Multi-fluid Model

2.1 Characteristics for Six Equation Model

The characteristics can be obtained by expanding continuity equations and rearranging them with the dependent variables vector, $\phi = \left(\alpha_g, \alpha_l, P, \upsilon_g, \upsilon_l, \upsilon_d\right)^T$,

$$\left(\rho_l \rho_g \frac{\alpha_d}{c_d^{\,2}} + \rho_d \rho_g \frac{\alpha_l}{c_f^{\,2}} + \rho_d \rho_l \frac{\alpha_g}{c_g^{\,2}}\right) \times$$
$$\left(\lambda-\upsilon_g\right)^2\left(\lambda-\upsilon_l\right)^2\left(\lambda-\upsilon_d\right)^2 -$$
$$\alpha_g \rho_l \rho_d \left(\lambda-\upsilon_l\right)^2\left(\lambda-\upsilon_d\right)^2 -$$
$$\alpha_l \rho_g \rho_d \left(\lambda-\upsilon_d\right)^2\left(\lambda-\upsilon_g\right)^2 -$$
$$\alpha_d \rho_g \rho_l \left(\lambda-\upsilon_g\right)^2\left(\lambda-\upsilon_l\right)^2 = 0 \qquad (1)$$

where c_g, c_l and c_d are sound speed for individual phases. α and υ are volume fraction and velocity respectively. Subscripts, g, l and d stands for gas, liquid and drop fields respectively.

Characteristic equation for three field 6 equation model is very similar to that of 4 equation model [4,10]. If the drop volume fraction is set to zero, one can get the characteristic equation for 4 equation model. We note that for the cases of practical interest, the liquid density is much higher than the vapor density. Then it is reasonable to neglect the terms with ρ_g and two real roots are obtained, that are approximately:

$$\lambda = \upsilon_g \pm c_g \qquad (2)$$

Generally, these real characteristics can be expressed as,

$$\lambda = \overline{\upsilon} \pm \overline{c} \qquad (3)$$

where $\overline{\upsilon}$ is a certain averaged material velocity, and $\overline{c}$ is a certain averaged sound velocity. And rest of the characteristics are two pair of complex conjugate values.

In RELAP5, one as well as four equation models have been thoroughly investigated. Von Neumann stability analysis [3] is utilized for their arguments. In this section, these steps are reviewed to check their applicability to SPACE code.

2.2 One Equation Model

The essential problems having CC can be well exposed with relatively simple one equation model [4]. In the four equation model, if $\upsilon_g, \upsilon_l \ll c_g$, complex roots are approximately

$$\mu \cong \frac{\upsilon_l + \varepsilon^2 \upsilon_g}{1+\varepsilon^2} \pm \frac{i\varepsilon}{1+\varepsilon^2}\left(\upsilon_l - \upsilon_g\right) \qquad (4)$$

where $\varepsilon^2 \equiv \left(1-\alpha_g\right)\rho_g \big/ \alpha_g \rho_l$.

Even though this approximation is for four equation case, it is also valid for six equation case when volume fraction of droplet approaches zero. The effect of complex characteristic velocities on finite-difference equations can be shown by reviewing a simple one equation model as follows:

$$\frac{\partial \phi}{\partial t} + \upsilon\left(1+\tau i\right)\frac{\partial \phi}{\partial z} + K\phi = 0 \qquad (5)$$

We assume that υ, τ and K are nonnegative constants. The traveling wave solution (dispersion analysis) gives the following:

$$\upsilon = \upsilon_0 \exp\left(\left(i\left(kx + k\upsilon t\right) - \left(K - k\upsilon\tau\right)t\right)\right) \qquad (6)$$

This gives unstable behavior if $K < k\upsilon\tau$, i.e., friction factor is lower than some value determined by the complex characteristics.

Let this equation be approximated by the difference equation,

$$\frac{\phi_j^{n+1} - \phi_j^n}{\Delta t} + \upsilon(1+\tau i)\frac{\phi_j^{n+1} - \phi_j^n}{\Delta z} + K\phi_j^{n+1} = 0 \qquad (7)$$

where subscript j refers to spatial node and n indicates time step. This difference equation uses donor-cell differencing for the explicit convective term and implicit treatment of the damping term, $K\phi$.

Applying the von Neumann linear stability analysis method [3], we can get the growth factor:

$$\lambda = (1+K\Delta t)^{-1}\left[1-\frac{\upsilon\Delta t}{\Delta z}(1+\tau i)\big(1-\exp(-ik\Delta z)\big)\right] \quad (8)$$

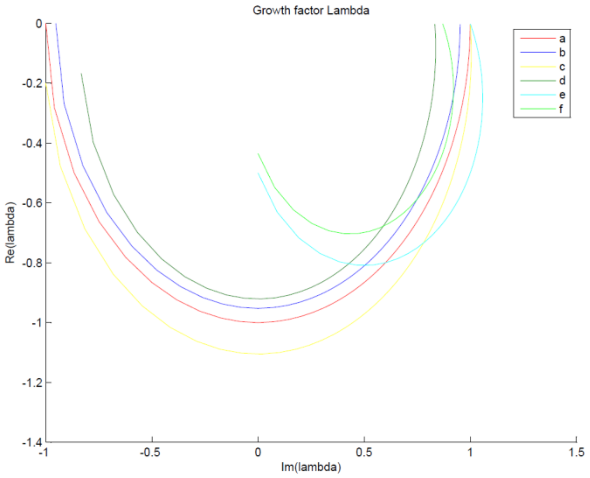

Fig. 1. Growth factor for one equation model

If $|\lambda|>1$, then the corresponding Fourier component of ϕ can grow geometrically, while if $|\lambda|\le 1$ for all k, the solution remains bounded for all time steps.

When $K,\tau=0$, the locus of values of λ for all m is a sequence of points on a circle in the complex plane with radius of Courant number ($CR \equiv \upsilon\Delta t/\Delta z$) centered at 1-$CR$ (a in Fig. 1.). If CR <1, it lies inside the unit circle, i.e., stable. If K >0, the radius is reduced so that it lies inside a unit circle (b). Now if $K=0$ but $\tau\neq 0$, then the previous locus is effectively dilated by $(1+\tau^2)^{1/2}$ and rotated by an angle $\arctan(\tau)$, both about the point 1 in the complex plane (c and e): then, no matter how small, there is an m large enough that the corresponding λ will be tilted outside the unit circle (even if CR <1; unstable). Finally, if $K>0$, then, the above locus is further changed by a contraction toward the origin $(1+K\Delta t)^{-1}$. Evidently, if K is large enough, the entire locus may lie inside the unit circle even for large m (d and f), and, become stable again.

2.3 Six Equation Model

Regularization of four equation case has been studied for the numerical as well as the differential approaches [10]. Dispersion and Von Neumann stability analysis for the six equation model are performed in this paper.

As can be seen in Fig. 2, typical annular mist flow regime shows unstable behavior (a), where typical interfacial drag force correlation is used. This happens in four equation model too [10]. Increasing drag force by factor of 1000 (c) guarantees the stability. Even 100 (b) times may work.

But one should not take those number too seriously because there are many other important aspects that are not included in this study. For example, entrainment/de-entrainment is the one.

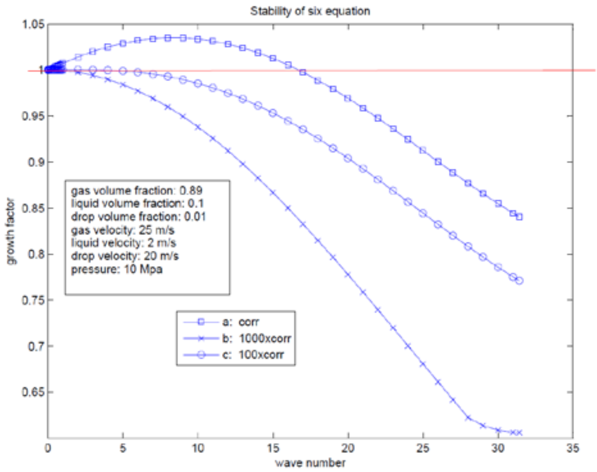

Fig. 2. Growth factor for six equation model

3. Conclusions

Careful reviews on the stability analysis for the multi-fluid model have been executed. One equation model is very useful to understand the behavior of the complex characteristics.

And the hydraulic solver for SPACE code seems to be as stable as that of RELAP5. Further detailed study should be performed for the final confirmation. The energy method or equivalent method have to be studied as well.

REFERENCES

[1] Ishii M., T. Hibiki, 2006, "Thermo-fluid dynamics of two-phase flow," Springer Science & Business Media Inc., New York.

[2] S. Y. Lee, T. Hibiki, M. Ishii, 2009 NED, Formulation of time and volume averaged two-fluid model considering structural materials in a control volume ", Vol-239 pp 127-139

[3] R. D. RICHTMYER AND K. W. MORTON, "Difference Methods for Initial Value Problems," Second Edn., Interscience, New York, 1967.

[4] H. B. STEWART, "Stability of Two-Phase Flow Calculations Using Two-Fluid Models," J. Comput. Phys., 33, 259, (1979).

[5] V. H. RANSOM and D. L. HICKS, "Hyperbolic Two Pressure Models for Two-Phase Flow," J. Comput. Phys., 53, 124, (1984).

[6] H. B. STEWART and B. WENDROFF, "Two-Phase Flow: Models and Methods," J. Comput. Phys., 56, 363, (1984).

[7] J. D. Ramshaw and J. Trapp, "Characteristics, stability and short wavelength phenomena in two-phase flow equation system," Nucl. Sci. Eng. 66, 93. (1978).

[8] RELAP5/MOD3.3 Code manual, volume vi: validation of numerical techniques in Relap5/Mod3.0, Nuclear Safety Analysis Division, Nureg/cr-5535/Rev-1, December 2001.

[9] S. J. Ha et al, "Development of the SPACE Code for Nuclear Power Plants," Nucl. Eng. and Tech., Vol. 43 No.1 February 2011

[10] H. Pokharna, M. Mori, V. Ransom, "Regularization of Two-Phase Flow Models: A Comparison of Numerical and Differential Approaches", J. Comput. Phys., 134, 363, 282-295, (1997).