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Parameter estimation using Bayesian updating method

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1. Introduction

One of the severe problems, environmentally assisted cracking (EAC) has been studied intensively during several decades, but its mechanisms are not well defined because of its detection. Therefore it is necessary to predict it using probabilistic estimation. For this reason, RI-ISI in a PWR is being spotlighted as excellent concept of inspection and maintenance recently. However in probabilistic estimation, there are too many parameters that have uncertainties. It is very important to decrease uncertainties of parameters and Bayesian parameter updating can be applied to decrease them using probability of detection. In this paper, parameter in initial crack depth distribution is estimated using Bayesian updating method. Also, this method is verified by comparison of errors. It is step required to predict some variables such as crack growth rate. Furthermore, it will contribute to reduce the risk of PWR.

2. Methods and Results

In this section, Bayes' theorem [1] is introduced briefly and the parameter in initial crack depth distribution is updated using Bayesian method by computational software MATLAB. After used assumptions are explained, simulation results follow.

2.1 Bayes' theorem

The principle of Bayes' theorem is conditional probability. Equation (1) shows it.

$$f(\theta|x) = \frac{gL(x|\theta)\,f(\theta)}{\int_{\varphi} g(x|\theta)f(\theta)d\theta} \qquad (1)$$

To conclude, unknown parameter θ is updated using known data x. In this equation f(θ) is defined as prior distribution for unknown parameter θ. Also, $g(x|\theta)$ in numerator is likelihood and it is conditional probability of known data x. Finally $f(\theta|x)$ is defined as posterior distribution. The denominator term is just regarded as normalizing constant because integral value of posterior distribution should be one.

The prior distribution in a Bayesian analysis usually embodies a subjective notion of probability since the distribution of a parameter such as θ rarely is known. It is the degree of belief about θ before the observational data x are obtained.

2.2 Assumptions

To estimate initial crack depth distribution, some assumptions are required [2]. At first, it is necessary to decide prior distribution of parameter. According to Marshall Report, crack depth distribution is exponential distribution.

$$p(a) = \lambda\exp(-\lambda a) \qquad (2)$$

In this distribution there is an unknown parameter λ, and it can be updated using Bayes' theorem. The prior distribution of λ is gamma distribution that is conjugate with likelihood, because it is convenient to update prior distribution.

$$\mathrm{pri}(\lambda) = b(\lambda b)^{k-1}\frac{\exp(-\lambda b)}{\Gamma(k)} \qquad (3)$$

In equation (3), b and k are constants to decide distribution scale and shape respectively.

Likelihood is expressed like equation (4).

$$L(\lambda|a) = \prod_{i}^{n} C_d(\lambda)p(a_i)\mathrm{POD}(a_i) \qquad (4)$$

The likelihood includes the information of crack depth distribution and Probability of detection (POD). It is known that likelihood is expressed by multiplying them when there is some information. C_d is normalizing constant.

2.3 Results

Considering above assumptions, Updating λ is done by equation 5. 10 data used in likelihood are adopted.

$$\mathrm{post}(\lambda) = C_{\lambda}L(\lambda|a)\mathrm{pri}(\lambda) \qquad (5)$$

Figure 1 shows distribution of prior, likelihood, posterior. It represents posterior distribution has narrower shape than prior distribution. The more it is updated; the variance of distribution will decrease. It means uncertainty of distribution is decreased. Then updated λ is substituted to equation (2) using Monte Carlo simulation. Random sampling of λ is done 10^5 times, and 10^5 initial crack depths are sampled too.

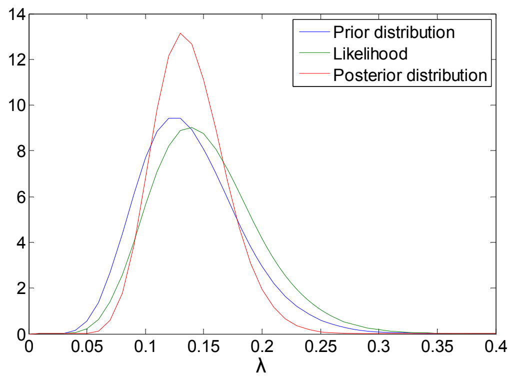

Figure 1. λ distributions

As a result, standard deviation of posterior initial crack depth is smaller than that of prior one. Also, the estimation of error between prior initial crack depth and posterior initial crack depth is performed. Equation (6) informs difference between predicted crack depths and observed data. It is similar to standard deviation.

$$\varepsilon = \sum_{i=1}^{n} \frac{(a_i - \bar{a}')^2}{n} \qquad (6)$$

In this equation, n is the number of random sampling and $\bar{a}'$ is average observed data used when λ is updated. In this simulation, n is 10^5 and it is supposed that $\bar{a}'$ is 7.23. The results are showed at Table I.

Table I. Comparison of distribution after updating with distribution before updating

	Before updating	After updating
Average crack depth	7.9949	7.5816
Error rate	10.58%	4.863%
Standard deviation	81.4893	63.2475
ε	80.9043	63.1240

As expected, updated a is closer to 7.23, observed a, and it is shown that the error is reduced after updating λ. It means uncertainties of both λ and a are reduced.

3. Conclusions

In nuclear power plants, aging problems about passive components are being treated as a hot issue recently. There are some limitations to estimate degradation modes of nuclear reactor using deterministic methods. Therefore instead of deterministic approach probabilistic approach should be developed. However, probabilistic approach is not prepared to use in many nuclear power plants.

Before using probabilistic approach to estimate reactor safety, it is needed to decrease uncertainties of parameters by some methods. Bayesian updating method can be answer when there is some information. This approach has some disadvantages that it is subjective because many assumptions are needed. However it is appropriate to increase quality of the inference and it can decrease not only standard deviation of unknown parameter but error with observed data.

In case of initial crack depth, prior distribution of unknown parameter λ is assumed as gamma distribution. Likelihood includes initial crack depths and those probably of detection. Then posterior distribution of λ is obtained. In other words, λ is updated using observed data, a'. After that, predicted crack depth, a will become close to observed data.

These results depend on many functions such as POD. Therefore it is valuable to update parameters using other POD. Also after initial crack depth is obtained, it is possible to use it to probabilistic approach of failure. If the stress is analyzed, the key values such as probability of failure can be found. Then it can be used in probability safety assessment (PSA). Aging PSA is able to be used for not only active components but passive components like pipe, tube.

REFERENCES

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