# Performance Test of System Identification Methods for a Nuclear Reactor

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## 1. Introduction

An automatic controller that uses the model predictive control (MPC) method is being developed for automatic load follow operation [1]. As described in Ref. [1], a system identification method is important in the MPC method because MPC is based on a system model produced by system identification.

There are many models and methods of system identification. In this study, AutoRegressive eXogenous (ARX) model was selected from among them, and the recursive least square (RLS) method and least square (LS) method associated with this model are used in a comparative performance analysis.

#### 2. Least Square Method of ARX Model

The ARX model is a suitable model for linear control implementations. The parameter estimation problem in the ARX model is convex and easily handed for both Single-Input/Single-Output (SISO) and Multi-Input/Multi-Output (MIMO) systems, in contrast to other models. The ARX model is shown in Eq.(1).

A(d)y(t) = B(d)u(t) + e(t)(1) Assuming that the measured error (e(t)) is negligible, Eq.(1) can be expressed in Eq.(2).

$$y(t) = ym(t) = \frac{B(d)}{A(d)}u(t)$$
  
$$rm(t) = u^{T}(t)\theta(t) \quad t = 1 \qquad N \qquad (2)$$

 $ym(t) = u^{T}(t)\theta(t), t = 1, ..., N$  (2) Here, ym(t) is the model output, u(t) is a vector of known quantities and  $\theta(t)$  is a vector of unknown quantities. The elements of vector u(t) are termed regression variables or regressors, and vector  $\theta(t)$  is known as a parameter vector. To estimate  $\theta(t)$  for given measurements of y(1), u(1),...,y(N),u(N), an error equation is introduced :

$$\varepsilon(t)=y(t)-ym(t)=y(t) - u^{T}(t)\theta(t), t=1,...,N$$

or, compactly

 $\epsilon$ =Y-Ym=Y - u<sup>T</sup> $\theta$  (3) In the least square method,  $\theta$  is chosen such that  $\epsilon^{2}(t)$  is small for all t:

$$\theta_{LS} = \frac{1}{2} \varepsilon^{T} \varepsilon = \frac{1}{2} (Y - u^{T} \theta)^{T} (Y - u^{T} \theta)$$

Assuming that  $u^T u$  is invertible, the solution of the optimization is given by solving  $\frac{\partial}{\partial \theta} \theta_{LS} = 0$ , which leads to

$$\theta_{\rm LS} = (\mathbf{u}^{\rm T} \mathbf{u})^{-1} \mathbf{u}^{\rm T} \mathbf{Y} \tag{4}$$

In general, the optimized parameters, as predicted by the proposed model, are different depending on the data set, such as u and Y.

#### 3. Recursive Least Square Method of ARX Model

Recursive system identification is a term used for estimation algorithms in which the estimated parameters are updated for each new observation. It relies on fast algorithms where the computation burden and required memory do not increase with time. It is also a key tool for handling cases in which the system dynamics vary over time. In this study, a recursive system identification method known as the recursive least square method is adopted. Using the recursive least square method, Eq.(2) can be written recursively, as follows:

$$\begin{aligned} \theta(t)_{RLS} &= \theta(t-1)_{RLS} + P(t)u(t)\{y(t) - u^{T}(t)\theta(t-1)_{RLS}\} \end{aligned} (5) \\ P(t) &= P(t-1) - \frac{P(t-1)u(t)u^{T}(t)P(t-1)}{1+u(t)P(t-1)u^{T}(t)} \end{aligned}$$

This algorithm (Eq.(5)) is known as the recursive least square algorithm. It requires very little memory and with low computational complexity and no matrix inversion. For every new step, only the matrix P needs to be updated.

## 4. Numerical Simulation

In NPPs, it is meaningless to generate a timeinvariant core model because the reactor core is very dynamic, complex, and has many disturbances. The best means of modeling the reactor core may be to recalculate a core model every time step. Consequently, in this study, model parameters of a reactor core are produced using the ARX model at every time step. The least square and recursive least square methods are used in conjunction with this.

The predicted outputs in the ARX models are the reactor power and the axial power shape, which should be controlled for a load-follow operation. The input values are control rod positions, including the Part Strength Control Element Assembly (PSCEA) and Full Strength Control Element Assembly (FSCEA) in addition to the reactor power and the axial power shape. Therefore, the ARX model is composed of a MIMO component and uses tenth-order past data. In addition, the KISPAC-1D code [2], which is capable of analyzing the reactor core and system in APR1400, is utilized to provide past data with the ARX model as an actual

plant. To simulate the transient reactor core for a loadfollow operation, control rods are artificially inserted and withdrawn. The detailed control change rates according to the simulation time are shown in Table 1.

Fable 1. Control rods char	iges for simulation
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Time	Change rates(cm/s)
$10s \sim 210s$	PSCEA(-1.0), FSCEA(-0.9)
210~ 410s	PSCEA(0.95), FSCEA(0.85)
410 ~ 610s	PSCEA(-1.25), FSCEA(-0.45)
610 ~ 810s	PSCEA(1.2), FSCEA(0.35)
$810 \sim 1000s$	PSCEA(-1.15), FSCEA(-1.0)
$1000 \sim 4000s$	Sine function according to time

## 5. Results

Figures 1, 2, 3 and 4 are simulation results. In all figures, black lines are the target outputs, in this case the power and axial shape index (ASI), and the red dots denote the predicted power and ASI outputs using the LS and the RLS methods.



Figure 1. Predicted power using the LS algorithm







Figure 3. Predicted ASI using the LS algorithm



Figure 4. Predicted ASI using the RLS algorithm

According to the figures, in general, the LS method compares poorly to the RLS method. In particular, the LS method leads to a considerable difference between the predicted outputs and the target values when the control rods are changed. The two methods show different results due to the differences between the LS and RLS algorithm. First, the LS algorithm cannot determine an optimized model when the matrix  $\mathbf{u}^{T}\mathbf{u}$  is singular. During the LS simulation, a singular matrix appeared several times. Second, the LS algorithm uses only tenth-order past data at time t. In contrast to the LS algorithm, the RLS algorithm uses past data and correction terms which depend on the prediction error. Therefore, the RLS method can calculate prediction values very precisely. The results of the RLS method are shown in Figure 2 and Figure 4. They show that the prediction values are nearly identical to the target values after periodic repetitions of the insertion and withdrawal of the control rods.

#### 6. Conclusion

A comparative evaluation between the LS and RLS methods was performed using the KISPAC-1D code. According to the simulation results, the RLS method is more accurate when the nuclear reactor power shows a regularly repeating pattern. Therefore, the RLS method is recommended for a daily load-follow operation in NPPs.

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### REFERENCE

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