Discrete Ordinates Method-Like Computation with Group Condensation and Angle Collapsing in Transport Theory

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1. Introduction

In group condensation for transport method, it is well-known that angle-dependent total cross section is generated.[1,2] To remove this difficulty on angle-dependent total cross section, we normally perform the group condensation on total cross section by using scalar flux weight as used in neutron diffusion method. In this study, angle-dependent total cross section is directly applied to the discrete ordinates method. In addition, angle collapsing concept is introduced based on equivalence to reduce calculational burden of transport computation.

We also show numerical results for a heterogeneous 1-D slab problem with local/global iteration, in which fine-group discrete ordinates calculation is used in local problem while few-group angle collapsed discrete ordinates calculation is used in global problem iteratively.

2. Methods and Results

2.1 Group Condensation

To get fine-group to few-group condensed cross sections, the fine-group discrete ordinates equation is considered first:

$$\mu_n \frac{\partial \psi_{n,g}(x)}{\partial x} + \sigma_g(x)\psi_{n,g}(x) = \sum_{l=0}^{\infty}(2l+1)P_l(\mu_n)\sum_{g'}\sigma_{l\,gg'}(x)\phi_{l\,g'}(x) + \frac{\chi_g}{k_{eff}}\sum_{g'}\upsilon\sigma_{f\,g'}(x)\phi_{g'}(x), \qquad (1)$$

where n indicates discrete angle index, g for fine-group energy structure.

If we sum eq.(1) over the energy within few-group ($g \in G$), then few-group discrete ordinates equation is obtained:

$$\mu_n \frac{\partial \psi_{n,G}(x)}{\partial x} + \hat{\sigma}_{n,G}(x)\psi_{n,G}(x) = \sum_{l=0}^{\infty}(2l+1)P_l(\mu_n)\sum_{G'}\sigma_{l\,GG'}(x)\phi_{l\,G'}(x) + \frac{\chi_G}{k_{eff}}\sum_{G'}\upsilon\sigma_{f\,G'}(x)\phi_{G'}(x), \qquad (2)$$

where

$$\hat{\sigma}_{n,G}(x) \equiv \frac{\sum_{g\in G}\sigma_g(x)\psi_{n,g}(x)}{\psi_{n,G}(x)}, \quad \sigma_{l\,GG'}(x) \equiv \frac{\sum_{g'\in G'}\sum_{g\in G}\sigma_{l\,gg'}(x)\phi_{l\,g'}(x)}{\phi_{l\,G'}(x)},$$
$$\upsilon\sigma_{f\,G}(x) \equiv \frac{\sum_{g\in G}\upsilon\sigma_{f\,g}(x)\phi_g(x)}{\phi_G(x)}. \qquad (3)$$

As shown in eq.(3), the group condensed total cross section has angle dependency. Normally, to remove this angle dependency on total cross section, we use scalar flux as a weighting function instead of angular flux, or employ some higher order approximations such as the consistent P approximation and extended transport approximation. However, in this paper, angle-dependent total cross section is directly applied in the sweeping calculation. This can be done by giving different total cross section to each discrete angle, and the detailed equation is shown in Section 2.2 with angle collapsing concept.

2.2 Angle Collapsing

To reduce calculational burden with angle-dependent total cross section in the few-group calculation, angle collapsing to positive and negative directions is also considered in this paper.

If we sum eq.(2) over positive and negative angle ordinates to perform angle collapsing, then the result is shown as below:

$$\frac{\partial}{\partial x}[\overline{\mu}_G^{\pm}(x)\psi_G^{\pm}(x)] + \hat{\sigma}_G^{\pm}(x)\psi_G^{\pm}(x) = \sum_{l=0}^{\infty}(2l+1)P_l^{\pm}\sum_{G'}\sigma_{l\,GG'}(x)\phi_{l\,G'}(x) + \frac{\chi_G}{k_{eff}}\sum_{G'}\upsilon\sigma_{f\,G'}(x)\phi_{G'}(x), \qquad (4)$$

where

$$J_G^+(x) \equiv \sum_{n=1}^{N/2} w_n\mu_n\psi_{n,G}(x),\ J_G^-(x) \equiv \sum_{n=1}^{N/2} w_{N+1-n}\mu_{N+1-n}\psi_{N+1-n,G}(x),$$
$$\psi_G^+(x) \equiv \sum_{n=1}^{N/2} w_n\psi_{n,G}(x), \quad \psi_G^-(x) \equiv \sum_{n=1}^{N/2} w_{N+1-n}\psi_{N+1-n,G}(x),$$
$$P_l^+ \equiv \sum_{n=1}^{N/2} w_nP_l(\mu_n), \qquad P_l^- \equiv \sum_{n=1}^{N/2} w_{N+1-n}P_l(\mu_{N+1-n}),$$
$$\hat{\sigma}_G^+(x) \equiv \frac{\sum_{n=1}^{N/2} w_n\hat{\sigma}_{n,G}(x)\psi_{n,G}(x)}{\psi_G^{\pm}(x)}, \qquad (5)$$
$$\hat{\sigma}_G^-(x) \equiv \frac{\sum_{n=1}^{N/2} w_{N+1-n}\hat{\sigma}_{N+1-n,G}(x)\psi_{N+1-n,G}(x)}{\psi_G^{\pm}(x)}, \quad \overline{\mu}_G^{\pm}(x) \equiv \frac{J_G^{\pm}(x)}{\psi_G^{\pm}(x)}.$$

Here w_n is a quadrature weight corresponding to the discrete angle set (for 1D discrete ordinates set, sum of the quadrature weights is equal to 2).

All of the angle collapsed parameters can be determined by eq.(3) and eq.(5). Note that $\overline{\mu}_G^{\pm}(x)$ has spatial distribution and also it depends on energy group. The total cross section has angle dependency in eq.(4). Therefore, sweeping calculation procedure is different from the conventional one [1,2]. If we integrate eq.(4) over fine-mesh interval and use diamond difference scheme for spatial discretization, we can get sweeping equation as follows:

$$\psi^{\pm}_{i,G} = \left(\frac{\left|\bar{\mu}^{\pm}_{i-1/2,G} + \bar{\mu}^{\pm}_{i+1/2,G}\right|}{2\left|\bar{\mu}^{\pm}_{i\pm1/2,G}\right|} \psi^{\pm}_{i\mp1/2,G} + \frac{q^{\pm}_{i,G} h_i}{2\left|\bar{\mu}^{\pm}_{i\pm1/2,G}\right|} \right) \times \left(1 + \frac{\hat{\sigma}^{\pm}_{i,G} h_i}{2\left|\bar{\mu}^{\pm}_{i\pm1/2,G}\right|} \right)^{-1}, \qquad (6)$$

where i denotes fine-mesh index, $i \pm 1/2$ for fine-mesh boundaries, and $q^{\pm}_{i,G}$ indicates “directional” average source determined by scattering and fission.

2.3 Numerical Results

A heterogeneous 1D slab problem is considered in this study. The problem configuration is shown in Fig.1.

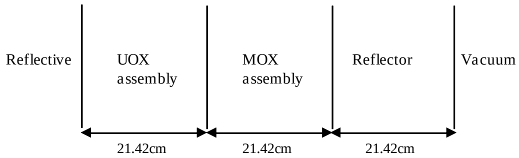

Fig. 1: Problem configuration

The inside of UOX/MOX assembly is heterogeneous, and detailed configuration is described in [3].

The seven group cross sections are given in the C5G7 benchmark problem report [3] as fine-group structure with isotropic scattering. S_{12} discrete ordinates are used for fine group calculations with an iteration criterion 10^{-7}.

In numerical calculation, 0.03cm very fine mesh is used and group condensed cross sections are generated for each calculation mesh to minimize discretization effect. The local/global iteration framework [4] is also tested until 1^{st} local/global iteration.

Table 1 shows k_{eff} values and its relative errors when angle-dependent total cross section is applied and conventional scalar flux weighted total cross section is used. In both cases, the angle collapsing concept is used.

Table 1: k_{eff} and relative error[a]

Local/Global iteration : 0^{th}		
	k_{eff}	Relative Error
Scalar flux weighted total XS	1.110347	918.5 pcm
Angle-dependent total XS	1.107532	662.7 pcm
Local/Global iteration : 1^{st}		
	k_{eff}	Relative Error
Scalar flux weighted total XS	1.105797	502.4 pcm
Angle-dependent total XS	1.100489	19.1 pcm

[a] Reference k_{eff} = 1.100241 from fine-group calculation

It shows better performance when angle-dependent total cross section is used for the problem tested. Figs.2 and 3 show that relative errors in scalar flux distribution, and the case with angle-dependent total cross section gives more accurate scalar flux distribution.

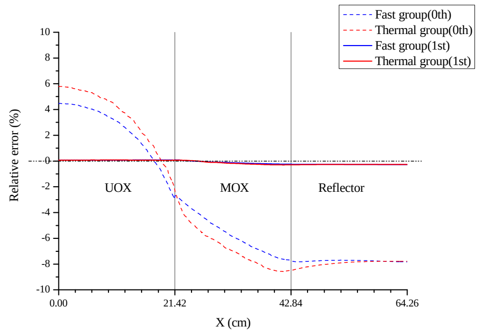

Fig.2: Relative error of scalar flux for application of angle-dependent total cross section and angle collapsing

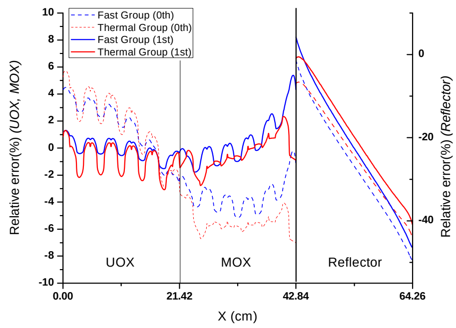

Fig.3: Relative error of scalar flux when scalar flux weighted total cross section is applied

3. Conclusions

Group condensed angle-dependent total cross section is applied to discrete ordinates equation without approximation in this study. An angle collapsing concept to positive and negative direction is also proposed.

The angle collapsing concept reduces calculational burden in angle-dependent total cross section few-group calculation. The method was used in local/global iteration framework for whole-core transport calculation, and the 1^{st} iteration gives excellent results for the 1-D slab problem tested.

Application to multi-dimensional problems should be straightforward.

REFERENCES

[1] G.I. Bell, S. Glasstone, *Nuclear Reactor Theory*, Litton Educational Publishing, Inc., (1970).

[2] E.E. Lewis, W.F. Miller, Jr., *Computational Methods of Neutron Transport*, John Wiley & Sons, Inc., (1984).

[3] E.E. Lewis *et al.*, Benchmark on Deterministic Transport Calculations Without Spatial Homogenisation : Appendix A, OECD/Nuclear Energy Agency, (2003).

[4] N.Z. Cho, S. Yun, J. Lee, Overlap Local/Global Iteration Framework for Monte Carlo/Diffusion Nodal Calculations, *Trans. Am. Nucl. Soc.* Vol.102 p.528, (2010).