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A comparative study of the lattice Boltzmann and volume of fluid method for the rising bubble flows

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1. Introduction

Recently, the lattice Boltzmann method (LBM) has gained much attention for its ability to simulate fluid flows, and for its potential advantages over a conventional CFD method. The key advantages of LBM are, (1) suitability for parallel computations, (2) absence of the need to solve the time-consuming Poisson equation for a pressure, and (3) an ease with the way multiphase flows, complex geometries and interfacial dynamics may be treated[1]. Nevertheless, the LBM is considered as a mere alternative CFD tools, not a promising approach.

The motion of the bubbles in a liquid has been the focus of both academic and practical interest. The central problem is the relationship between the rise velocity, bubble shape due to the interface deformation and flow field. The buoyancy effect due to density difference in the two phase flows is characterized with Eotvos and Morton numbers[2].

In this study, a single bubble rising under a buoyancy is simulated with LBM and VOF based on conventional CFD method. The two simulation results are compared with the previous experiments. The main objective of the present work is to establish the lattice Boltzmann method as a viable tool for the simulation of multiphase or multi-component flows.

2. Methods and Results

2.1 VOF method

Here, we consider a flow with two phases which have different densities. The low density and high density are noted as ρ_L and ρ_H respectively. The flow can be described by the Navier-Stokes equations and a volume of fluid equation as [3]

$$\frac{\partial F}{\partial t} + \nabla \cdot (F\vec{u}) = 0 \quad (1)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\vec{u}) = 0 \quad (2)$$

$$\frac{\partial \rho\vec{u}}{\partial t} + \nabla \cdot (\rho\vec{u}\vec{u}) = -\nabla p + \mu\nabla^2\vec{u} + \vec{F}_b \quad (3)$$

where $\vec{F}_b$ is the body force, and ρ , μ are defined as

$$\rho = F\rho_A + (1-F)\rho_B , \quad \mu = F\mu_A + (1-F)\mu_B$$

where ρ_A and ρ_B are the density of fluid A and fluid B respectively, and F is the volume fraction of fluid.

2.2 Lattice Boltzmann method

Here, we consider a Cahn-Hilliard equation instead of Eq. (1). The remainings are same. The flow can be described by the Navier-Stokes equations and an interface evolution equation as [4]

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi\vec{u}) = \theta_M \nabla^2 \mu_\phi \quad (4)$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{u}) = 0 \quad (5)$$

$$\frac{\partial n\vec{u}}{\partial t} + \nabla \cdot (n\vec{u}\vec{u}) = -\nabla \cdot P + \mu\nabla^2\vec{u} + \vec{F}_b \quad (6)$$

where θ_M is called mobility, μ_ϕ is the chemical potential, P is the pressure tensor, $\vec{F}_b (= \phi\vec{g}, if \quad \phi < 0)$ is the body force, and n, ϕ are defined as

$$n = \frac{\rho_A + \rho_B}{2}, \phi = \frac{\rho_A - \rho_B}{2}$$

where ρ_A and ρ_B are the density of fluid A and fluid B respectively.

Under the lattice Boltzmann framework, Eq. (4) can be solved by iterating the evolution equation for a set of distribution functions. These distribution functions evolve with a modified lattice Boltzmann equation and BGK approximation,

$$g_i(x + \vec{e}_i\delta t, t + \delta t) = g_i(x,t) + \Omega_i + (1-q)\delta g_i$$

with

$$\Omega_i = \frac{g_i^0(x,t) - g_i(x,t)}{\tau_\phi}$$

$$\delta g_i = g_i(x + \vec{e}_i\delta t, t) - g_i(x,t) \quad (7)$$

where g_i is the distribution function, Ω_i is the collision term, τ_ϕ is the dimensionless single relaxation time, $\vec{e}_i$ is the lattice velocity, and q is a constant coefficient.

In Eq. (6), the term $\nabla \cdot P$ is related to the surface tension force. This force can be rewritten as a potential term,

$$\vec{F}_s = -\nabla \cdot P = -\phi\nabla\mu_\phi - \nabla p_0$$

where $p_0 = nc_s^2$, c_s is the speed of sound.

The potential form for the surface tension force is adopted to keep the energy conservation. Mathematically, the potential form and stress form are identical. However, numerically, the discretization error is different[4]. Thus, it is useful to eliminate spurious currents.

The lattice Boltzmann implementation of Eqs. (5) and (6) can be described as

$$f_i(x+\vec{e}_i\delta t, t+\delta t) = f_i(x,t) + \Omega_i \qquad (8)$$

with

$$\Omega_i = \frac{f_i^0(x,t) - f_i(x,t)}{\tau_n} +$$

$$(1-\frac{1}{2\tau_n})\frac{w_i}{c_s^2}[(\vec{e}_i - \vec{u}) + \frac{(\vec{e}_i \cdot \vec{u})}{c_s^2}\vec{e}_i](-\phi\nabla\mu_\phi + \vec{F}_b)\delta t$$

The equilibrium distributions satisfy the conservation laws as

$$\phi = \sum_i g_i, n = \sum_i f_i$$

$$\vec{u} = [\sum_i f_i\vec{e}_i + \frac{1}{2}(-\phi\nabla\mu_\phi + \vec{F}_b)]/n$$

The details are Ref. [4].

2.3 Results

The two dimensional single bubble rising under a buoyancy is simulated. The density ratio is 1000. The bubble is surrounded with stationary walls. Initially, it is located at a lower region of the computational domain (80x300). The dimensionless parameters are defined as

$$Eo = \frac{g(\rho_H - \rho_L)D^2}{\sigma}, \quad M = \frac{g(\rho_H - \rho_L)\mu_H^4}{\rho_H^2\sigma^3}.$$

The bubble will rise at a nearly constant velocity due to the balance between the buoyancy and the drag force. The comparison of simulation results are shown in Fig. 1. The two present results are in good agreement with the previous experiments. As shown in Fig. 1, the Eotvos number(*Eo*) increases gradually from 5 to 240. The increase of *Eo* is equivalent to the decrease of the surface tension. These will enhance the deformation of a bubble. The VOF model do not reduce the spurious currents of the static droplet test at the acceptable level and do not satisfy with the Laplace law for the small droplet. The VOF results are agreed well with the experiments in case of low *Eo* number. For high *Eo* number, the VOF results are deviated from the experiments and the LBM numerical results. These results are clearly presented in Fig. 2.

3. Conclusion

The lattice Boltzmann method for two phase flows has been applied to the simulations of bubbles under a buoyancy. The results for the rise velocity, and the bubble shapes with Eotvos and Morton numbers were found to be in good agreement with the VOF method and LBM method in case of a high surface tension(low *Eo*). For the low surface tension(high *Eo*), the LBM results are better than the VOF method.

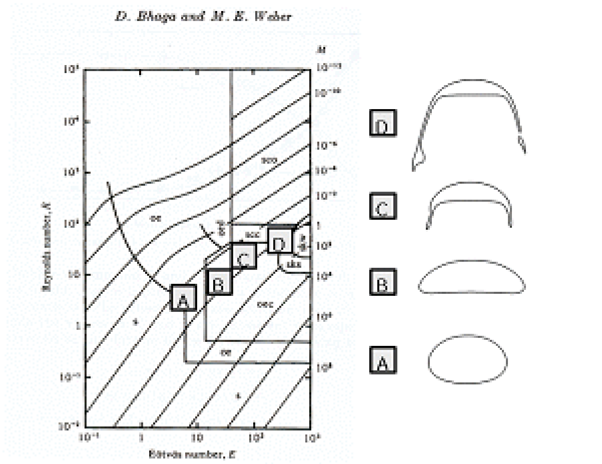

Fig. 1 The flow regime map of experiments(2): S, Spherical; OE, oblate ellipsoid; OED, oblate ellipsoidal (disk-like and wobbling); OEC, oblate ellipsoidal cap; SCC, spherical cap with closed, steady wake; SCO, spherical cap with open, unsteady wake; SKS, skirted with smooth, steady skirt; SKW, skirted with wavy, unsteady skirt.; VOF numerical results A B C D.

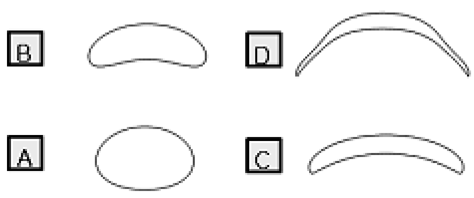

Fig. 2 The final bubble shapes of the LBM results.

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