Uncertainty Analysis of a Support Vector Regression Model for Estimating Collapse Moment of Wall-Thinned Pipes

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1. Introduction

The wall-thinned defect is mainly caused by flow-accelerated corrosion, and it decreases failure pressure, load-carrying capacity, deformation ability, and fatigue resistance of pipes. Thus it is necessary to examine the effect of wall-thinned defects on the failure behavior of pipes and to accurately estimate the collapse loads of wall-thinned pipes under various loading conditions. This work incorporates the support vector regression (SVR) that has been successfully employed to solve nonlinear regression and time series forecasting problems. To solve the support vectors, the collapse moment-related data should be provided. These data were obtained by performing finite element analyses (FEAs) for various loading conditions and defect geometries such as the thinning defect locations of extrados, intrados and crown, bend radius, bend angle, wall thickness at the thinning defect, thinning length, thinning angle, internal pressure, and bending modes of closing and opening. The collapse moment was predicted using these loading conditions and defect geometries as the inputs into the SVR models.

2. Support Vector Regression

2.1 Model Development

An SVR model searches for the network weights of an artificial neural network with a kernel function by solving the non-convex unconstrained minimization problem. The hypothesis space of the linear functions is performed using an SVR model in multidimensional feature space. The basic concept of SVR is to nonlinearly map the original input data $\mathbf{x}$ into high dimensional feature space $\boldsymbol{\varphi}$ and then to conduct linear regression in the feature space. The unknown regression function can be solved by determining the coefficients of the basis function of linear expansion. The support vector approximation is expanded as follows:

$$y = f(\mathbf{x}) = \sum_{k=1}^{N} w_k \phi_k(\mathbf{x}) = \mathbf{w}^T \boldsymbol{\varphi}(\mathbf{x}) + b \qquad (1)$$

The function $\phi_k(\mathbf{x})$ is called the feature and the parameters $\mathbf{w}$ and b are the support vector weight and bias, respectively, which are calculated by minimizing the following regularized risk function [1]:

$$R(\mathbf{w}) = \frac{1}{2}\mathbf{w}^T\mathbf{w} + \frac{\lambda}{h}\sum_{k=1}^{N}\left|y_k - f(\mathbf{x})\right|_{\varepsilon}^{h} \qquad (2)$$

The constrained optimization problem can be solved by applying the Lagrange multiplier technique. The regression function of Eq. (2) can be expressed as follows:

$$y = f(\mathbf{x}) = \sum_{k=1}^{N}\gamma_k \boldsymbol{\varphi}^T(\mathbf{x}_k)\boldsymbol{\varphi}(\mathbf{x}) + b = \sum_{k=1}^{N}\gamma_k K(\mathbf{x},\mathbf{x}_k) + b \qquad (3)$$

where $K(\mathbf{x},\mathbf{x}_k) = \boldsymbol{\varphi}^T(\mathbf{x}_k)\boldsymbol{\varphi}(\mathbf{x})$ is called the kernel function. A number of coefficients $\alpha_i - a_i^*$ are nonzero values and the corresponding training data points have approximation error equal to or larger than ε . They are called support vectors. The bias is calculated as

$$b = -\frac{1}{2}\sum_{k=1}^{N}\gamma_k\left(K(\mathbf{x}_r,\mathbf{x}_k) + K(\mathbf{x}_s,\mathbf{x}_k)\right) \qquad (4)$$

where $\mathbf{x}_r$ and $\mathbf{x}_s$ are support vectors (SVs) and these are data points outside the ε -insensitivity zone.

2.2 Selection of Training Data

The SC scheme introduces the concept of the information potential to determine the quantity of the information. Each data point is considered as a potential cluster center. The information potential of each data point is defined as

$$P_1(k) = \sum_{j=1}^{N} e^{-4\left\|\mathbf{x}_k - \mathbf{x}_j\right\|^2 / r_\alpha^2}, \; k = 1, 2, ..., N, \qquad (5)$$

In general, after the i th cluster center has been determined, the potential of each data point is revised using the following equation:

$$P_{i+1}(k) = P_i(k) - P_i^c e^{-4\left\|\mathbf{x}_k - \mathbf{c}_i\right\|^2 / r_\beta^2}, \; k = 1, 2, ..., N, \qquad (6)$$

These calculations stop if the inequality $P^*(i) < \varepsilon P^*(1)$ becomes true, otherwise calculation continues. The input/output data positioned in cluster centers are selected to train the SVR model.

3. Uncertainty Analysis

By using an uncertainty analysis, a prediction interval can be calculated such that the exact value exists in the prediction interval at a specified confidence level. In this paper, an analytic uncertainty analysis method was used.

The regression models of Eq. (3) can be expressed as

$$y_k = f(\mathbf{x}_k, \boldsymbol{\theta}) + \varepsilon_k \qquad (7)$$

For a regression model of an observation $\mathbf{x}_o$ which is not part of the training data, the output prediction is given by the following:

$$\hat{y}_0 = f(\mathbf{x}_0, \hat{\boldsymbol{\theta}}) \quad (8)$$

The output prediction can be approximated according to the Taylor series expansion of the output prediction to the first order as follows:

$$\hat{y}_0 \approx f(\mathbf{x}_0, \boldsymbol{\theta}) + \mathbf{f}_0^T \cdot \left[\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\right] \quad (9)$$

Then the prediction error can be calculated using the following:

$$y_0 - \hat{y}_0 = \varepsilon_0 - \mathbf{f}_0^T \cdot \left[\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\right] \quad (10)$$

If the parameter is presumed to be estimated explicitly with the well-known squared error minimization technique, the variance-covariance matrix can be estimated as follows:

$$\mathbf{S} = s^2 \left(\mathbf{F}^T \mathbf{F}\right)^{-1} \quad (11)$$

The matrix **F** is called the Jacobian matrix of first order partial derivatives about the parameters determined from the least squares.

The variance of the predicted output can be estimated as follows [2]:

$$Var(y_0 - \hat{y}_0) \approx \sigma^2 + \mathbf{f}_0^T \mathbf{S} \mathbf{f}_0 \approx s^2 + s^2 \mathbf{f}_0^T \left(\mathbf{F}^T \mathbf{F}\right)^{-1} \mathbf{f}_0 \quad (12)$$

The estimate with a 95% confidence interval is

$$\hat{y}_0 \pm 2s\sqrt{1 + \mathbf{f}_0^T \left(\mathbf{F}^T \mathbf{F}\right)^{-1} \mathbf{f}_0} = \hat{y}_0 \pm \delta \quad (13)$$

4. Application to the Collapse Moment Estimation

The characteristic of the collapse moment is much different according to the three wall-thinned defect locations of extrados, intrados, and crown. Therefore, the data are classified into the three classes of defect locations and three SVR models are designed for the three classes, respectively. Table 1 summaries the estimation results of collapse moments and coverage of the prediction intervals.

Table 1: Estimation results of the collapse moment and coverage of the prediction interval

Defect location	Test data			
	No.	RMS Error (%)	Data number exceeding prediction interval	Coverage Probability (%)
Extrados	170	1.6867	7/170	95.88%
Intrados	170	0.6855	8/170	95.29%
Crown	32	0.5300	5/32	84.38%
Total	372	0.9674	20/372	94.62%

For the test data, the root mean square (RMS) error is 0.9674% and the collapse moment at 20 data point among 372 test data points exceeded the predicted interval, indicating 94.62% coverage. In addition, among three cases of the wall-thinned defect locations, the simulation results of only the intrados case are represented in the Fig. 1 and Fig. 2.

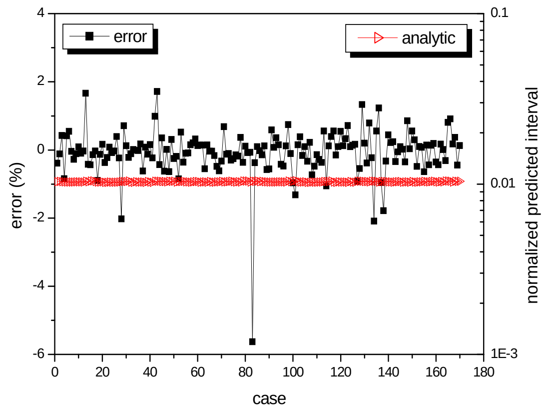

Fig. 1 The errors between actual collapse moment and estimated one for the test data and prediction intervals

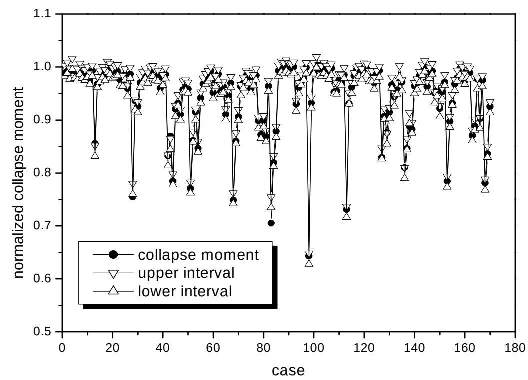

Fig. 2 Coverage of prediction intervals for the test data

5. Conclusions

In this paper, an SVR model has been developed to estimate the collapse moment of wall-thinned pipes. In addition, prediction intervals were calculated using an analytic uncertainty analysis. The RMS error is 0.9674% for the test data. In addition, it is known from the simulation results that prediction intervals are very narrow, which mean that the predicted values are accurate. Thus if the SVR models are optimized by using a number of data including a variety of defect geometry cases and loading conditions, they can accurately estimate the collapse moment for any other defect cases.

REFERENCES

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