Optimization Algorithms for Nuclear Reactor Power Control

Yeong-Min Kim[a*], Won-Jong Oh[a], Seung-Jin Oh[a], Won-Gee Chun[a], Yoon-Joon Lee[a]
[a]*Department of Nuclear and Energy Engineering, Jeju National University, Korea*
**km820426@jejunu.ac.kr*

1. Introduction

One of the control techniques that could replace the present conventional PID controllers in nuclear plants is the linear quadratic regulator (LQR) method. The most attractive feature of the LQR method is that it can provide the systematic environments for the control design. However, the LQR approach heavily depends on the selection of cost function and the determination of the suitable weighting matrices of cost function is not an easy task, particularly when the system order is high. The purpose of this paper is to develop an efficient and reliable algorithm that could optimize the weighting matrices of the LQR system.

2. Optimization Goal and Methods

The overall configuration of the digital reactor power control system is illustrated in Fig. 1. The details regarding the physical backgrounds and the reactor modeling are described in Ref. [1]. The power control system is a tracking system in which the output follows a command signal, and by augmenting the integrated error signal, the system is recast into the order increased regulating system.

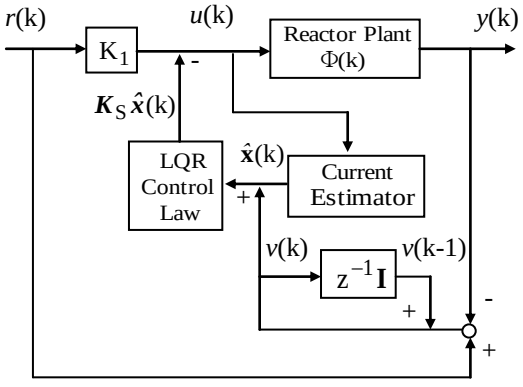

Fig. 1. Configuration of the reactor power control system

The system equation is

$$\boldsymbol{\xi}(k+1) = \boldsymbol{\Psi}\boldsymbol{\xi}(k) + \boldsymbol{\Lambda} r(k), \quad \boldsymbol{y}(k+1) = \boldsymbol{H}\boldsymbol{\xi}(k) \qquad (1)$$

where $\boldsymbol{\xi}$ is the state variable vector and $\boldsymbol{\Psi}$, $\boldsymbol{\Lambda}$, $\boldsymbol{H}$ are system matrices. The optimal performance function of the order increased LQR is

$$J_S = \frac{1}{2}\sum_{k=0}^{\infty}(\boldsymbol{x}(k)^T \boldsymbol{Q}\boldsymbol{x}(k) + u(k)^T R u(k)) \qquad (2)$$

where $\boldsymbol{x} = (\hat{\boldsymbol{x}}(k) \quad v(k))$, and $\boldsymbol{Q}$, R are the weighting matrices, respectively.

The design purpose of the order increased LQR system is to determine the state feedback gain $\boldsymbol{K}_S$, which, in turn, is determined by the weighting matrices of $\boldsymbol{Q}$ and R. Since the goal was to determine the optimal weighting matrix $\boldsymbol{Q}$ in a stochastic way, the problem is formulated simply as follows:

$$\textit{Find} \quad \boldsymbol{X} = [x_1, x_2, ..., x_6],$$

$$\textit{To Minimize Cost } (\boldsymbol{X}) = \sum_{k=0}^{\infty} | y(k) - y_0 | + \sum_{k=0}^{\infty} | u(k) | \qquad (3)$$

where $x_i \equiv$ diagonal elements of the weighting matrix $\boldsymbol{Q}$.

As is in Eq. (3), it is very difficult to describe the cost function *Cost* ($\boldsymbol{X}$) and its Jacobian and Hessian precisely as a function of $\boldsymbol{X}$. The conventional gradient-based searching methods are not suitable and other stochastic searching techniques should be considered.

2.1. Determination of Control Parameters by SA

The SA (Simulated Annealing) is efficient for the combinatorial optimization, especially when the objective function cannot be expressed analytically. The main concepts are the cooling schedule and the neighborhood generation mechanism.

Once a current solution $\boldsymbol{X}$ has been obtained, a new solution $\boldsymbol{X}'$ called *neighbor* is generated from the current solution $\boldsymbol{X}$ through a proper neighborhood generation mechanism. Whenever the neighbor $\boldsymbol{X}'$ decreases the cost (i.e., $\Delta C = Cost(\boldsymbol{X}') - Cost(\boldsymbol{X}) < 0$), it is accepted as a new current solution. Even when the neighbor makes the cost increase, it is also possible that it can be accepted if the probability of the acceptance, $Prob = exp(-\Delta C/T)$, is greater than a random number chosen between 0 and 1 at that instance. Here, ΔC is the cost change and T is the annealing parameter equivalent to the temperature in the physical annealing process [2].

2.2. Determination of Control Parameters by MGA

The GA (Genetic Algorithm) has proven to be a useful tool in a variety of search and optimization problems [3]. It does not depend on the coupling of design parameters and can escape from local traps. But it has demerits of wandering around the true solution without further convergence, resulting in poor performance. The traditional GA is improved by complementing several modified

schemes, and is named the modified GA (MGA). The major modifications include: (1) linear fitness scaling, (2) elite policy, (3) variable number of crossover site, (4) modified crossover scheme, (5) ancestor-pool and (6) periodic re-initialization of the population.

An exponential-wise representation is devised as $x_i = a_i \cdot e^{b_i}$, $i = 1, 2, ..n$. This kind of the individual representation is very useful either when there is little information about the parameter range. A modified crossover scheme instead of the well-known bit-wise crossover is developed to avoid the hamming-cliff effects. To save the execution time, the fitness calculation is dropped if the values of the design parameters coincide with those of the previously visited solution. And by introducing a pre-specified size of storage pool named hereafter an *ancestor-pool*, some amount of the total execution time could be reduced. The elite-policy is often employed to retain good solutions. However, the dense application of the elite-policy may lead to a pre-maturing result. By using the ancestor-pool in the MGA, all the individuals in the population are re-initialized periodically from the up-to-best solutions stored in the ancestor-pool. This makes the searching process more stable and it accelerates the improvements of the solutions.

2.3. Combination of the MGA with the SA

The SA is a one-point searching scheme and suitable to locate the final solution. However, its performance in the improvement of the solution is generally rather slow at the earlier stage of the searching process compared with the GA. On the other hand, the GA is a multi-point searching scheme and thus more efficient than the SA for the global search in the earlier searching stage. But the GA also has its own drawback in that it tends to wander around the true solution at the final stage of the searching process due to this multi-point searching scheme. Thus, it is expected to improve the performance through the combination of these two methods. By doing so, the GA could compensate the drawbacks of the SA or vice versa in such a way that: Firstly, perform the stochastic random but evolutionary search by using the MGA for the large feasible domain until a solution of pre-specified quality is found. Then, start the SA to locate a final solution with the MGA's result as an initial solution.

3. Application Results and Conclusions

To determine the proper weighting matrices of the LQR system by using the MGA, the problem is set forth as:

$$\textit{Find } \boldsymbol{C} = [a_1, b_1, a_2, b_2, \ldots, a_6, b_6]$$

$$\textit{To maximize fitness}(\boldsymbol{X}) = 1/\textit{Cost}(\boldsymbol{X})$$

$$\textit{Subject to Cost}(\boldsymbol{X}) = \sum_{k=0}^{\infty} |y(k) - y_0| + \sum_{k=0}^{\infty} |u(k)|,$$

$$\boldsymbol{X} = [x_1, x_2, \ldots, x_6], \quad x_i = a_i \cdot e^{b_i}, \; i = 1, 2, \ldots, 6 \quad (4)$$

where $\boldsymbol{C} \equiv$ chromosome of an individual (or a candidate solution) and others are the same as in Eq.(3).

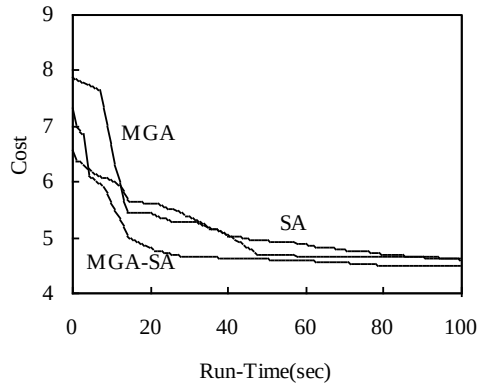

Fig. 2. Cost values of SA, MGA and MGA-SA

Fig. 2 shows the performances of the algorithms discussed so far. It shows that the hybrid MGA-SA shows the superior results both in the run-time and the minimization of the cost function. Fig. 4 describes the responses of the system with the weighting functions determined by MGA-SA. The power is step increased by 10 % from the initial state of 90% power. As is shown, the peak value of the output is sufficiently lower than the value of 103% which is set forth in the Final Safety Analysis Report (FSAR).

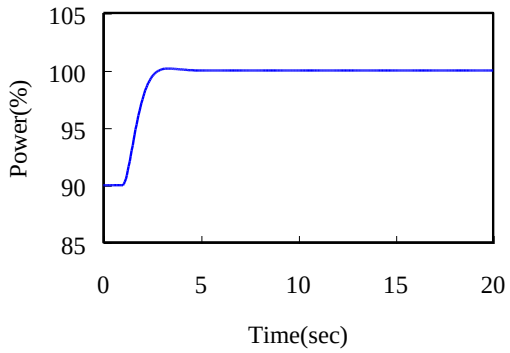

Fig. 3 Output responses of OIRS

Various stochastic searching algorithms are applied to the determination of the weighting parameters of the LQR reactor power control system. These stochastic searching methods give a systematic design procedure and yield good control characteristics.

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