Simulation of Time-Dependent P_3 Equations Using a Semi-Analog Medium

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1. Introduction

A wide variety of numerical methods have been introduced to solve the neutron transport equation for reactor calculations. With the state-of-the-art computer technology, successful implementation of higher-order approximation of transport methods (P_N, S_N, MOC, etc.) may now be feasible. Although these methods have been adaptable to code parallelization techniques, the computational expense remains a significant obstacle and thwarts their implementation in a whole-core, time-dependent methodology. A novel method to remove this problem is based on the method of cellular neural networks (CNN) coupling with the P_N method. Parallel data processing in CNN reduces the processing time and makes it possible to solve the time dependent models of neutron transport equation in real time.

2. Theory

2.1 CNN Architecture

The CNN architecture has previously been discussed thoroughly [1]. We, however, now go ahead to modify the equivalent electrical circuit of each cell to model the second-order form of the P_3 equations using CNN. Fig. 1 shows a new cell electrical circuit. The second-order nonlinear differential equation defining the dynamics of each cell can be derived as follows:

$$C\frac{dv_{xij}}{dt} = -\frac{1}{R_x}v_{xij} + \sum_{C(k,l)\in N_r(i,j)} A(i,j:k,l)v_{yij} + I_s - D_{gain}\frac{d^2v_{xij}}{d^2t}, 1\le i\le M, 1\le j\le N \quad (1)$$

2.2 Neutron Transport Equation

To obtain the second-order form of time dependent neutron transport equation we use spherical harmonic expansion of angular flux as:

$$\psi_g(\vec{r},\hat{\Omega}) = \sum_{n=0}^{\infty}(2n+1)\times\sum_{m=0}^{n}\phi_{nmg}(\vec{r})cosm\varphi + \gamma_{nmg}(\vec{r})sinm\varphi]P_{nm}(cos\theta) \quad (2)$$

where θ and φ are axial and azimuth angles. $P_{nm}(cos\theta)$ is the associated Legendre polynomial. The application of a Galerkin scheme to the neutron transport equation results in a set of first-order partial differential equations. The first-order partial differential equations allow odd moments (n = odd) to be expressed in terms of the even moments and their derivatives. If these expressions for the odd moments are substituted throughout equations, the resulting system is obtained as a set of second-order partial differential equations [2]. The second-order form of the P_3 approximation to the neutron transport in the *x-y* geometry is given below for an isotropic source and a homogeneous case.

$$\frac{1}{v}(2\sigma_t-\sigma_s-\nu\sigma_f)\frac{\partial\phi_{00}}{\partial t} = -\frac{1}{v^2}\frac{\partial^2\phi_{00}}{\partial^2 t} + \frac{1}{3}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\phi_{00} - \frac{1}{3}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\phi_{20} + 2\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)\phi_{22} + 4\frac{\partial^2\gamma_{22}}{\partial x\partial y} - \sigma_t(\sigma_t-\sigma_s)\phi_{00} + \sigma_t\frac{1}{k_{eff}}\nu\sigma_f\phi_{00} + \sigma_t q^e \quad (3\text{-a})$$

$$\frac{1}{v}(2\sigma_t)\frac{\partial\phi_{20}}{\partial t} = -\frac{1}{v^2}\frac{\partial^2\phi_{20}}{\partial^2 t} + \frac{5}{21}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\phi_{20} - \frac{1}{15}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\phi_{00} - \frac{4}{7}\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)\phi_{22} - \frac{8}{7}\frac{\partial^2}{\partial x\partial y}\gamma_{22} - \sigma_t^2\phi_{20} \quad (3\text{-b})$$

$$\frac{1}{v}(2\sigma_t)\frac{\partial\phi_{22}}{\partial t} = -\frac{1}{v^2}\frac{\partial^2\phi_{22}}{\partial^2 t} + \frac{3}{7}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\phi_{22} + \frac{1}{30}\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)\phi_{00} - \frac{1}{21}\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)\phi_{20} - \sigma_t^2\phi_{22} \quad (3\text{-c})$$

$$\frac{1}{v}(2\sigma_t)\frac{\partial\gamma_{22}}{\partial t} = -\frac{1}{v^2}\frac{\partial^2\gamma_{22}}{\partial^2 t} + \frac{3}{7}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\gamma_{22} + \frac{1}{15}\frac{\partial^2}{\partial x\partial y}\phi_{00} - \frac{2}{21}\frac{\partial^2}{\partial x\partial y}\phi_{20} - \sigma_t^2\gamma_{22} \quad (3\text{-d})$$

In spherical harmonic approximation of the neutron transport equation we use the following form of vacuum boundary condition in steady state:

$$\int_{\vec{n}.\vec{\Omega}<0}\vec{n}.\vec{\Omega}P_{nm}(cos\theta)\sin m\varphi\,\psi(\vec{r},\hat{\Omega},E) = 0 \quad (4\text{-a})$$

$$\int_{\vec{n}.\vec{\Omega}<0}\vec{n}.\vec{\Omega}P_{nm}(cos\theta)\cos m\varphi\,\psi(\vec{r},\hat{\Omega},E) = 0 \quad (4\text{-b})$$

where n is even and $n < N$, $0 \le m \le n$, and $\vec{r} \in \Gamma$.

By using the same process as has been used by Inanc, the following equation gives the P_3 boundary condition for the right side of a rectangular region [2]. Similar equations for the top, left and bottom sides can be determined. At reflected boundaries the moment is either zero or has zero gradients depending on whether the spherical harmonic has odd or even parity there.

$$3\sigma_t\phi_{00} + 2\frac{\partial\phi_{00}}{\partial x} = 0 \quad (5\text{-a})$$

$$150\sigma_t\phi_{20} + 91\frac{\partial\phi_{20}}{\partial x} = 0 \quad (5\text{-b})$$

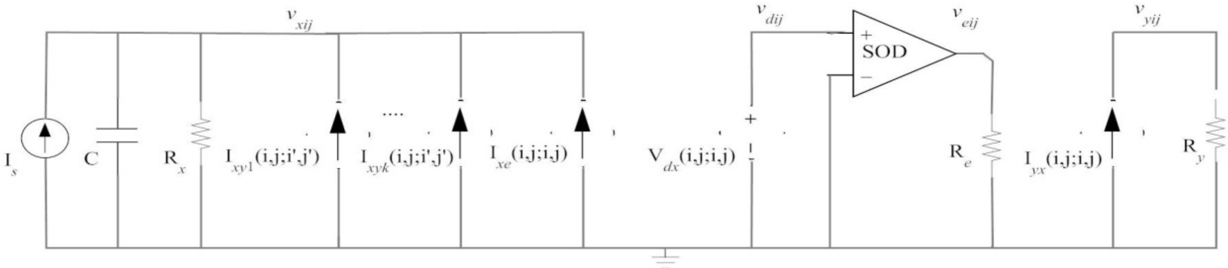

Fig. 1. New cell circuit where SOD is a second-order differentiator.

$$192\sigma_t\phi_{22} + 245\frac{\partial\phi_{20}}{\partial x} = 0 \qquad (5\text{-c})$$

$$35\sigma_t\gamma_{20} + 24\frac{\partial\gamma_{20}}{\partial x} = 0 \qquad (5\text{-d})$$

3. CNN Model for P_3 Equations

To obtain the CNN model for P_3 equation we first discretize Eq. (3) using finite difference approximation. The equivalent electrical elements for each equation are then obtained by comparing the resulting equations with Eq. (1) [1]. Finally a two-dimensional (2D) 4-layer CNN is required to simulate the P_3 equations. The CNN model being complex enough, we must use computer programs to study the transient behavior. In this study, the HSPICE software which is a professional electrical circuit package is used to simulate the CNN model.

4. Results and Conclusions

To verify the CNN results and show the ability of CNN in computing of the neutron flux distribution in steady state and transient conditions in the core, we examine a mini boiling water reactor (BWR) assembly problem. This problem is a simplified 2D simulation of a BWR fuel assembly, the surrounding coolant, and a portion of a fully inserted cruciform control rod. One quarter of the control rod is modeled. Fig. 2 illustrates a model that is decomposed into uniform 0.4 cm square cells. The one-group material properties for each of the three regions in this problem are listed in Table I [3]. CASMO-4 was employed by Taylor so as to generate a reference solution for this assembly configuration. The CASMO-4 reference effective multiplication factor k_{eff}

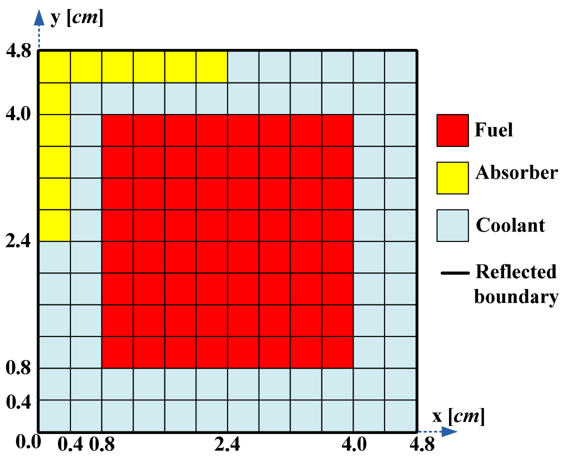

Fig. 2. Problem Schematic for 2D Mini-BWR Assembly.

Table I: Material Properties for 2D Mini-BWR Assembly

Region	σ_t [cm^{-1}]	σ_a [cm^{-1}]	$\nu\sigma_f$ [cm^{-1}]	σ_s [cm^{-1}]
Fuel	0.436526	0.0251889	0.0350929	0.411337
Coolant	0.487382	0.00254071	0.0	0.484841
Absorber	0.643697	0.1	0.0	0.543697

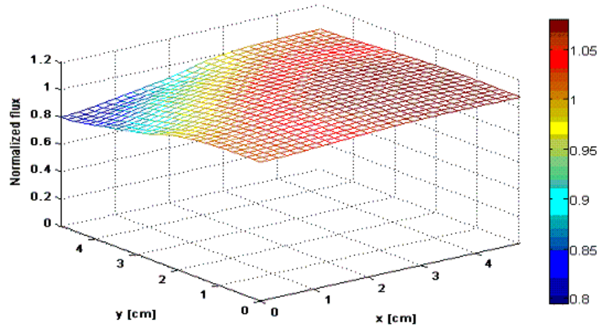

Fig. 3. Normalized scalar flux distribution for mini-BWR assembly calculated by CNN.

is calculated to be 0.82904.

To simulate this problem using CNN, the Cartesian space is discretized into 30×30 nodes. Then the CNN model elements are computed. The CNN calculation yields k_{eff} of 0.82647, which differs from the reference value by -0.31%. The steady state neutron scalar flux distribution is shown in Fig 3. The current results are comparable to Tylor's computed using the method of characteristic (MOC) [3].

In this work we introduced a time-dependent second-order form of the P_3 approximation for the neutron transport equation. The CNN method was utilized to solve these equations. The computed results by the CNN model were concurred with the reference. The advantages of the CNN simulator over the numerical methods have to do with its analog and parallel processing algorithm which reduces the computational time [1].

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