Semi-Analysis for the Pseudo-Colloid Migration of Multi-member Decay Chains in the Fractured Porous Medium with the Flux Boundary

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1. Introduction

Far-field modeling of radionuclide transport is an important component of general safety assessment studies carried out within the framework of storage of high-level radioactive waste in underground repositories.

After a canister failure, radionuclides are leached from the backfilling and penetrate the surrounding bedrock, the final barrier between pollutant and Man's environment. Migration by pure diffusion through a hard tock or clay barrier is a rather slow process. In Fractured porous media, all of the groundwater flow occur within the fractures because fractures have permeabilities of several orders of magnitude larger than those of the rock matrix, if the geological layers are fully saturated with water. So radionuclides dissolved in groundwater will be transported along a fracture with molecular diffusion from the fracture to the rock matrix.

Molecular diffusion from the fractures into the porous matrix constitutes an attenuation mechanism that can be highly order to prepare for extreme cases, it is assumed that the pollutants arrive rapidly in a fractured zone where transport takes place at much higher velocities. The specific problem of radionuclide transport through a fractured medium has been tackled by many scientists.

According to the electromagnetic interaction between the solute and the colloid, solutes are absorbed by the colloid, and then we are called the pseudo-colloid.

The natural colloid can exist inside a fracture with a density of 10^5 particles per one liter of a liquid. When the radionuclide migrates through a fractured rock, solutes sorb on natural colloids as well as the stationary fracture wall solid surface. Due to natural colloids, whose particle size is larger than that of solutes, colloids can migrate faster than solutes. Therefore, these pseudo-colloids, which are the sorbed solute molecules on the natural colloids, can also migrate faster than the solute. Both the solute and the pseudo-colloid are sorbed onto and desorbed from stationary solid wall surface of a fracture. Even though the colloid can not penetrate into the surrounding rock matrix, the solute diffuses into the porous matrix.

In this analysis, the Laplace transformed solution is derived for a pseudo-colloid in a fracture and a solute in a fracture and a surrounding rock with a realistic inlet boundary condition. The canonical form solutions are used for the Fortran based a computational code which uses a special subroutine for the inversion of Laplace transform based on the Theory of Green. This new solution can be combined with the total system performance assessment code such as the Goldsim which can exchange the information such as the inlet concentration and the outlet flux with the newly developed code by using its dll file.
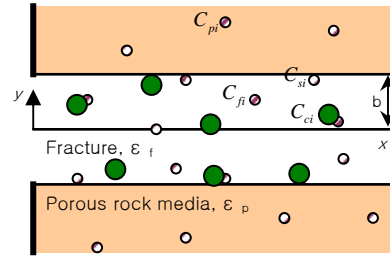
2. Mathematical Method

Fig. 1. The conceptual model in the fractured porous media.

It was considered a radioactive solute at concentration in the water in a thin fracture of the fractured porous rock. Also present in the fracture are the natural colloids, on which the solute can be sorbed to form the pseudo-colloid as shown in Fig. 1. It is assumed the one-dimensional convective-diffusive transport within the fracture, and that the colloids are too large to diffuse into the rock matrix. Neglecting the possible filtration within a fracture, the governing equation of the radionuclides sorbed on the colloid, pseudo-colloid, migration is:

$$\varepsilon_f \xi_l \frac{\partial C_{ci}(x,t)}{\partial t} + \varepsilon_f \xi_l v_c \frac{\partial C_{ci}(x,t)}{\partial x} - \varepsilon_f \xi_l D_c \frac{\partial^2 C_{ci}(x,t)}{\partial x^2}$$
$$+ \varepsilon_f \xi_l \lambda_i C_{ci}(x,t) - \varepsilon_f \xi_l \lambda_{i-1} C_{ci(-1)}(x,t) + \varepsilon_f S_1(x,t) + \varepsilon_f S_2(x,t) = 0$$
$$x > 0, \quad t > 0 \quad (1)$$

where ε_f is the ration of the liquid volume to total volume in the fracture, [-],

$C_{ci}(x,t)$ is the amount of species sorbed on the colloid per unit volume of the colloid, [kg/m^3],

v_c is the colloid pore velocity in the fracture, [m/yr],

D_c is the colloid dispersion coefficient, [m^2/yr]

λ_i is the decay constant of ith radionuclide, [1/yr],

ξ_l is the constant volume fraction of natural colloids in a liquid in the fracture, [-],

$\varepsilon_f S_1(x,t)$ is the rate of sorption of the pseudo-colloid to the stationary fracture wall, and $\varepsilon_f S_2(x,t)$ is the rate of desorption of the radionuclide from the pseudo-colloid.

Here, the concentration is assumed to be well mixed across the thin aperture of the fracture, 2b. Also the amount of the natural colloid is assumed to be so large compared with that of the pseudo-colloid that ξ_l is set to be constant.

For the same specie as a solute in the liquid in the fracture,

$$\varepsilon_f \frac{\partial C_{fi}(x,t)}{\partial t} + \varepsilon_f v_{si} \frac{\partial C_{fi}(x,t)}{\partial x} - \varepsilon_f D_{si} \frac{\partial^2 C_{fi}(x,t)}{\partial x^2} + \frac{q_i(x,t)}{b}$$
$$+ \varepsilon_f \lambda_i C_{fi}(x,t) - \varepsilon_f \lambda_{i-1} C_{fi(-1)}(x,t) - \varepsilon_f S_2(x,t) + \varepsilon_f S_3(x,t) = 0$$
$$x > 0, \quad t > 0 \quad (2)$$

where $C_{fi}(x,t)$ is the solute concentration in the liquid in the fracture, [kg/m^3],

v_{si} is the solute pore velocity, [m/yr],

D_{si} is the solute dispersion coefficient, [m^2/yr],

$\varepsilon_f S_3(x,t)$ is the rate of solute sorption on the stationary fracture wall, [kg/m^3·yr],

b is the fracture half-width, [m],

and $q_i(x,t)$ is the diffusive solute flux into the rock matrix, [kg/m^2·yr], given by

$$q_i(x,t) = -\varepsilon_p D_{pi} \frac{\partial C_{pi}(x,y,t)}{\partial y}\bigg|_{y=b} \quad (3)$$

where y is the distance from the center of the fracture, [m],

ε_p is the rock porosity, [-],

D_{pi} is the solute diffusion coefficient in the rock matrix pores, [m^2/yr],

and $C_{pi}(x,y,t)$ is the solute concentration in the pore water in the rock matrix, [kg/m^3],

$$R_{pi} \frac{\partial C_{pi}(x,y,t)}{\partial t} - D_{pi} \frac{\partial^2 C_{pi}(x,y,t)}{\partial y^2}$$
$$+ R_{pi} \lambda_i C_{pi}(x,y,t) - R_{p(i-1)} \lambda_{i-1} C_{p(i-1)}(x,y,t) = 0$$
$$x > 0, \quad y > 0, \quad t > 0 \quad (4)$$

where R_{pi} is the solute retardation coefficient in the rock matrix.

For solute species sorbed on the stationary fracture

$$(1-\varepsilon_f) \frac{\partial C_{si}(x,t)}{\partial t} - \varepsilon_f S_3(x,t)$$
$$+ (1-\varepsilon_f) \lambda_i C_{si}(x,t) - (1-\varepsilon_f) \lambda_i C_{si}(x,t) = 0$$
$$x > 0, \quad t > 0 \quad (5)$$

where $C_{si}(x,t)$ is the concentration of a sorbed solute on the stationary fracture.

For pseudo-colloids on the stationary fracture:

$$(1-\varepsilon_f)\xi_2 \frac{\partial C_{ci}(x,t)}{\partial t} - \varepsilon_f S_1(x,t)$$
$$+ (1-\varepsilon_f)\xi_2 \lambda_i C_{ci}(x,t) - (1-\varepsilon_f)\xi_2 \lambda_i C_{ci}(x,t) = 0$$
$$x > 0, \quad t > 0 \quad (6)$$

Where ξ_2 is the constant volume fraction of the sorbed colloid on the stationary fracture.

Linear sorption equilibrium between the solute species in the liquid in a fracture and the same species sorbed on the colloid is assumed. Both the solute species and the colloids in the liquid in a fracture are assumed to undergo the linear sorption equilibrium with the fracture solids:

$$K_{d1i} = \frac{\xi_2}{\xi_1}, \quad K_{d2i} = \frac{C_{si}}{C_{fi}}, \quad K_{d3i} = \frac{C_{ci}}{C_{fi}} \quad (7)$$

Adding equations (1) through (6) with equations (7), the equation for $C_{ci}(x,t)$ is obtained in terms of the effective retardation factor R_i, the dispersion coefficient D_i, and the velocity v_i.

$$R_i \frac{\partial C_{ci}(x,t)}{\partial t} + v_i \frac{\partial C_{ci}(x,t)}{\partial x} - D_i \frac{\partial^2 C_{ci}(x,t)}{\partial x^2}$$
$$+ R_i \lambda_i C_{ci}(x,t) - R_{i-1} \lambda_{i-1} C_{c(i-1)}(x,t) + \frac{q_i(x,t)}{\varepsilon_f b} = 0$$
$$x > 0, \quad t > 0 \quad (8)$$

The initial and boundary conditions are

$$C_{pi}(x,\infty,t) = 0, \qquad x > 0, \quad t > 0 \quad (9)$$

$$C_{pi}(x,b,t) = \frac{C_{ci}(x,t)}{K_{d3i}}, \qquad x > 0, \quad t > 0 \quad (10)$$

$$C_{pi}(x,y,0) = 0, \qquad x > 0, \quad y > 0 \quad (11)$$

$$A\left\{ v C_{ci}(0,t) - D \frac{\partial C_{ci}(x,t)}{\partial x}\bigg|_{x=0} \right\} = f_i(t), \ x > 0 \quad (12)$$

$$C_{ci}(\infty,t) = 0, \qquad t > 0 \quad (13)$$

$$C_{ci}(x,0) = 0, \qquad x > 0 \quad (14)$$

The semi-analytical solution of equations (8) and (9) to (14) can be found in Ahn's analysis on the solute only migration in a fractured porous media.

$$\tilde{C}_{ci}(x,s) = \sum_{j=1}^{i} u_{ji}(s) a_j(s) \exp[-b_j(s)x] \quad (15)$$

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