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Derivation of the Multi-fluid Model using the Time-Volume Averaging Method in Porous Body

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1. Time averaging operations

The local instantaneous balance equation reads as [1],

$$\frac{\partial}{\partial t}\left(\rho_k\psi_k\right)+\nabla\cdot\left(\rho_k\psi_k\mathbf{u}_k\right)=-\nabla\cdot\mathbf{J}_k+\rho_k\phi_k \tag{1}$$

where $\rho_k,\psi_k,\mathbf{u}_k,\mathbf{J}_k$ and ϕ_k are the density, property of extensive characteristics, velocity, flux and source of k-phase, respectively. Nomenclatures for other variables are found in [2]. Table-1 shows the field variables. $e_k,\mathbf{q}_k,\ \ p_k,\gamma_k,\mathbf{g}_k,\dot{q}_k,\mathbf{M}_k,E_k$ are internal energy, heat flux, pressure, vaporization, gravity and internal heat rate, interfacial momentum and energy sources respectively. $\mathbf{T}_k$ is the stress tensor and is decomposed into pressure and shear.

Table-1. Field variables

	ψ_k	$\mathbf{J}_k$	ϕ_k	I_k
Mass	1	0	0	γ_k
Momentum	$\mathbf{u}_k$	$-\mathbf{T}_k(p_k\mathbf{I}-\mathfrak{T}_k)$	$\mathbf{g}_k$	$\mathbf{M}_k$
Energy	$e_k+\mathbf{u}_k^2/2$	$\mathbf{q}_k-\mathbf{T}_k\cdot\mathbf{u}_k$	$\mathbf{g}_k\cdot\mathbf{u}_k+\dot{q}_k/\rho_k$	E_k

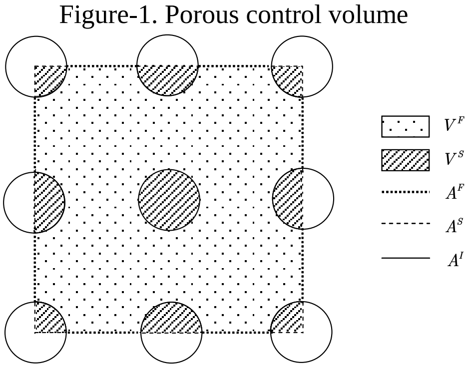
The time averaged balance equation for any property ψ_k of k-phase can be presented as,

$$\begin{aligned}&\frac{\partial}{\partial t}\left(\overline{\rho_k\psi_k}\right)+\nabla\cdot\left(\overline{\rho_k\psi_k\mathbf{u}_k}\right)=-\nabla\cdot\overline{\mathbf{J}_k}+\overline{\rho_k\phi_k}+I_k\\&I_k\equiv-\frac{1}{\Delta t}\sum_j\left\{\frac{1}{u_{ni}}\left[\left(\rho_k\psi_k\right)\mathbf{n}_k\cdot\left(\mathbf{u}_k-\mathbf{u}_i\right)-\left(\mathbf{n}_k\cdot\mathbf{J}_k\right)\right]\right\}\end{aligned} \tag{2}$$

where the bar over a quantity indicates the time-averaging operation. $\mathbf{n}$ is the surface normal vector in this paper. Considering the time-fluctuating terms, the time-averaged balance equation can be represented by using weighted mean variables as, ($\overline{\mathbf{J}_k^T}$; turbulent effects);

$$\frac{\partial}{\partial t}\left(\overline{\rho_k}\widehat{\psi_k}\right)+\nabla\cdot\left(\overline{\rho_k}\widehat{\psi_k}\widehat{\mathbf{u}_k}\right)=-\nabla\cdot\left(\overline{\mathbf{J}_k}+\overline{\mathbf{J}_k^T}\right)+\overline{\rho_k\phi_k}+I_k \tag{3}$$

Figure-1. Porous control volume



2. Local volume averaging operations

As shown in figure-1, V, A and ε indicate porosity, volume and area. Superscript T,F,S,I mean total, fluid, structure and internal, respectively[3].

$$V^T=V^F+V^S,\ \ A^T=A^F+A^S,\ \ \varepsilon^V\equiv V^F/V^T,\ \ \varepsilon^A\equiv A^F/A^T \tag{4}$$

Define phase average of scalar, vector or tensor $\mathcal{R}_k$,

$$\left[\mathcal{R}_k\right]^V\equiv\left(1/V^T\right)\int_{V^F}\mathcal{R}_k dV;\ \left[\mathcal{R}_k\right]^A\equiv\left(1/A^T\right)\int_{A^F}\mathcal{R}_k dA. \tag{5}$$

and define the intrinsic phase average,

$$[\![\mathcal{R}_k]\!]^V\equiv\left(1/V^F\right)\int_{V^F}\mathcal{R}_k dV;\ \ [\![\mathcal{R}_k]\!]^A\equiv\left(1/V^F\right)\int_{A^F}\mathcal{R}_k dA. \tag{6}$$

Absence of the field in structure yields the porosity;

$$\int_{V^F}\mathcal{R}_k dV=\int_{V^T}\mathcal{R}_k dV,\ \ \left[\mathcal{R}_k\right]^V=\varepsilon^V[\![\mathcal{R}_k]\!]^V \tag{7}$$

$$\int_{A^F}\mathcal{R}_k dA=\int_{A^T}\mathcal{R}_k dA,\ \ \left[\mathcal{R}_k\right]^A=\varepsilon^A[\![\mathcal{R}_k]\!]^A \tag{8}$$

Divergence theorem reads,

$$\int_{V^F}\nabla\cdot\mathcal{R}_k dV=\int_{A^F}\mathcal{R}_k\cdot\mathbf{n}^F dA+\int_{A^I}\mathcal{R}_k\cdot\mathbf{n}^I dA, \tag{9}$$

Divergence of an intrinsic local volume average;[2];

$$\nabla\cdot\left(\int_{V^F}\mathcal{R}_k dV\right)=\int_{A^F}\mathcal{R}_k\cdot\mathbf{n}^F dA. \tag{10}$$

Combining it with divergence theorem yields;

$$\int_{V^F}\nabla\cdot\mathcal{R}_k dV=\nabla\cdot\left(\int_{V^F}\mathcal{R}_k dV\right)+\int_{A^I}\mathcal{R}_k\cdot\mathbf{n}^I dA. \tag{11}$$

Time-volume averaged balance equation can be obtained by integrating (3) over V^F and dividing the integration by the volume element V^T as;

$$\left(1/V^T\right)\int_{V^F}\left(\frac{\partial}{\partial t}\left(\overline{\rho_k}\widehat{\psi_k}\right)+\nabla\cdot\left(\overline{\rho_k}\widehat{\psi_k}\widehat{\mathbf{u}_k}\right)+\nabla\cdot\left(\overline{\mathbf{J}_k}+\overline{\mathbf{J}_k^T}\right)-\overline{\rho_k\phi_k}-I_k\right)dV=0 \tag{12}$$

No-slip condition on the internal surface reads,

$$\frac{1}{V^T}\int_{A^I}\left(\overline{\rho_k}\widehat{\psi_k}\widehat{\mathbf{u}_k}\right)\cdot d\mathbf{A}^I=0 \tag{13}$$

$\mathbf{A}$ is the area normal vector. Individual terms yield;

$$\begin{aligned}&\frac{1}{V^T}\int_{V^F}\frac{\partial}{\partial t}\left(\overline{\rho_k}\widehat{\psi_k}\right)dV=\frac{\partial}{\partial t}\left[\frac{1}{V^T}\int_{V^F}\left(\overline{\rho_k}\widehat{\psi_k}\right)dV\right]=\frac{\partial}{\partial t}\left(\varepsilon^V[\![\overline{\rho_k}\widehat{\psi_k}]\!]^V\right)\\&\frac{1}{V^T}\int_{V^F}\nabla\cdot\left(\overline{\rho_k}\widehat{\psi_k}\widehat{\mathbf{u}_k}\right)dV=\nabla\cdot\left(\frac{1}{V^T}\int_{V^F}\overline{\rho_k}\widehat{\psi_k}\widehat{\mathbf{u}_k}dV\right)=\nabla\cdot\left(\varepsilon^V[\![\overline{\rho_k}\widehat{\psi_k}\widehat{\mathbf{u}_k}]\!]^V\right)\\&\frac{1}{V^T}\int_{V^F}\nabla\cdot\left(\overline{\mathbf{J}_k}+\overline{\mathbf{J}_k^T}\right)dV=\nabla\cdot\left[\frac{1}{V^T}\int_{V^F}\left(\overline{\mathbf{J}_k}+\overline{\mathbf{J}_k^T}\right)dV\right]+\frac{1}{V^T}\int_{A^I}\left(\overline{\mathbf{J}_k}+\overline{\mathbf{J}_k^T}\right)\cdot d\mathbf{A}^I\\&=\nabla\cdot\left[\varepsilon^V\left([\![\overline{\mathbf{J}_k}]\!]^V+[\![\overline{\mathbf{J}_k^T}]\!]^V\right)\right]+\frac{1}{V^T}\int_{A^I}\left(\overline{\mathbf{J}_k}+\overline{\mathbf{J}_k^T}\right)\cdot d\mathbf{A}^I\\&\frac{1}{V^T}\int_{V^F}\overline{\rho_k\phi_k}dV=\varepsilon^V[\![\overline{\rho_k\phi_k}]\!]^V.\\&\frac{1}{V^T}\int_{V^F}I_k dV=\varepsilon^V[\![I_k]\!]^V.\end{aligned} \tag{14}$$

Using the above results one can get;

$$\begin{aligned}&\frac{\partial}{\partial t}\left(\varepsilon^V[\![\overline{\rho_k}\widehat{\psi_k}]\!]^V\right)+\nabla\cdot\left(\varepsilon^V[\![\overline{\rho_k}\widehat{\psi_k}\widehat{\mathbf{u}_k}]\!]^V\right)=+\varepsilon^V[\![\overline{\rho_k\phi_k}]\!]^V\\&-\nabla\cdot\left[\varepsilon^V\left([\![\overline{\mathbf{J}_k}]\!]^V+[\![\overline{\mathbf{J}_k^T}]\!]^V\right)\right]-\frac{1}{V^T}\int_{A^I}\left(\overline{\mathbf{J}_k}+\overline{\mathbf{J}_k^T}\right)\cdot d\mathbf{A}^I+\varepsilon^V[\![I_k]\!]^V\end{aligned} \tag{15}$$

Define following weighted mean variables;

$$\left\langle\left\langle\widehat{\psi_k}\right\rangle\right\rangle^V\equiv\left([\![\alpha_k\overline{\overline{\rho_k}}\widehat{\psi_k}]\!]^V/[\![\alpha_k\overline{\overline{\rho_k}}]\!]^V\right)=[\![\alpha_k\widehat{\psi_k}]\!]^V/[\![\alpha_k]\!]^V \tag{16}$$

$$C_{\psi k}^V\equiv\left([\![\alpha_k\widehat{\psi_k}\widehat{\mathbf{u}_k}]\!]^V\right)\Big/\left([\![\alpha_k]\!]^V\left\langle\left\langle\widehat{\psi_k}\right\rangle\right\rangle^V\left\langle\left\langle\widehat{\mathbf{u}_k}\right\rangle\right\rangle^V\right) \tag{17}$$

$$\left\langle\left\langle\overline{\overline{\mathbf{J}_k}}\right\rangle\right\rangle^V\equiv[\![\alpha_k\overline{\overline{\mathbf{J}_k}}]\!]^V/[\![\alpha_k]\!]^V,\left\langle\left\langle\mathbf{J}_k^T\right\rangle\right\rangle^V\equiv[\![\alpha_k\mathbf{J}_k^T]\!]^V/[\![\alpha_k]\!]^V \tag{18}$$

$$[\![\alpha_k\overline{\overline{\rho_k\phi_k}}]\!]^V=\overline{\overline{\rho_k}}[\![\alpha_k]\!]^V[\![\widehat{\phi_k}]\!]^V \tag{19}$$

Finally one can get;

$$\frac{\partial}{\partial t}\left(\varepsilon^V[\![\alpha_k]\!]^V\overline{\overline{\rho_k}}\left\langle\left\langle\widehat{\psi_k}\right\rangle\right\rangle^V\right)+\nabla\cdot\left(C_{\psi k}^V\varepsilon^V[\![\alpha_k]\!]^V\overline{\overline{\rho_k}}\left\langle\left\langle\widehat{\psi_k}\right\rangle\right\rangle^V\left\langle\left\langle\widehat{\mathbf{u}_k}\right\rangle\right\rangle^V\right)$$

$$= -\nabla\cdot\left[\varepsilon^V\left([\![\alpha_k]\!]^V\left\langle\left\langle\overline{\overline{\mathbf{J}_k}}\right\rangle\right\rangle^V + [\![\alpha_k]\!]^V\left\langle\left\langle\overline{\overline{\mathbf{J}_k^T}}\right\rangle\right\rangle^V\right)\right]$$
$$-\frac{1}{V}\int_{A^I}\alpha_k\overline{\overline{\mathbf{J}_k}}\cdot d\mathbf{A}^I + \varepsilon^V[\![\alpha_k]\!]^V\overline{\overline{\rho_k}}[\![\widehat{\phi_k}]\!]^V + \varepsilon^V[\![I_k]\!]^V \tag{20}$$

3. Multi-fluid balance equations

Using the table-1, one can get mass balance equation;

$$\frac{\partial}{\partial t}\left(\varepsilon^V[\![\alpha_k]\!]^V\overline{\overline{\rho_k}}\right) + \nabla\cdot\left(\varepsilon^V[\![\alpha_k]\!]^V\overline{\overline{\rho_k}}\left\langle\left\langle\widehat{\mathbf{u}_k}\right\rangle\right\rangle^V\right) = \varepsilon^V[\![\gamma_k]\!]^V \tag{21}$$

Dropping out the averaging operators reads;

$$\frac{\partial}{\partial t}(\varepsilon\alpha_k\rho_k) + \nabla\cdot(\varepsilon\alpha_k\rho_k\mathbf{u}_k) = \varepsilon\gamma_k \tag{22}$$

Similar procedure with unity $C_{\mathbf{u}_k}^V$, the momentum balance equation is written as;

$$\frac{\partial}{\partial t}(\varepsilon\alpha_k\rho_k\mathbf{u}_k) + \nabla\cdot(\varepsilon\alpha_k\rho_k\mathbf{u}_k\mathbf{u}_k) = -\nabla\cdot(\varepsilon\alpha_k p_k\mathbf{I}) + \varepsilon\alpha_k\rho_k\mathbf{g}_k$$
$$+\nabla\cdot\left(\varepsilon\alpha_k\left(\mathcal{T}_k + \mathcal{T}_k^T\right)\right) - \left(1/V^T\right)\int_{A^I}\alpha_k\left(p_k\mathbf{I} - \mathcal{T}_k\right)\cdot d\mathbf{A}^I + \varepsilon\mathbf{M}_k, \tag{23}$$

Also, similar procedure with unity $C_{e_k}^V$, energy equation reads;

$$(\partial/\partial t)\left(\varepsilon\rho_k\alpha_k\left(e_k + \left(\mathbf{u}_k{}^2/2\right)\right)\right) + \nabla\cdot\left(\varepsilon\rho_k\alpha_k\left(e_k + \left(\mathbf{u}_k{}^2/2\right)\right)\mathbf{u}_k\right) =$$
$$+\varepsilon\rho_k\alpha_k\mathbf{g}_k\cdot\mathbf{u}_k - \nabla\cdot\left(\varepsilon\alpha_k\left(\mathbf{q}_k + \mathbf{q}_k^T\right)\right) - \nabla\cdot\left(\varepsilon\alpha_k\left(p_k\mathbf{I} - \mathcal{T}_k\right)\cdot\mathbf{u}_k\right) \tag{24}$$
$$-\left(1/V^T\right)\int_{A^I}\alpha_k\left(\mathbf{q}_k - \mathbf{T}_k\cdot\mathbf{u}_k\right)\cdot d\mathbf{A}^I + \varepsilon E_k$$

Momentum/energy sources are modeled as follows[1]

$$\mathbf{M}_k = \mathbf{M}_k^\gamma + p_{ki}\nabla\alpha_k + \mathbf{M}_{ik} - \nabla\alpha_k\cdot\mathcal{T}_{ki}, \tag{25}$$

$$E_k = \gamma_k\left(h_{ki} + \mathbf{u}_{ki}\cdot\mathbf{u}_k - \mathbf{u}_k{}^2/2\right) + a_i q_{ki}'' - p_{ki}(\partial/\partial t)\alpha_k$$
$$+\mathbf{u}_{ki}\cdot\mathbf{M}_{ik} - \nabla\alpha_k\cdot\mathcal{T}_{ki}\cdot\mathbf{u}_{ki} + W_{ki}^T \tag{26}$$

where W_{ki}^T is turbulent dissipation work. Subscript *ki* indicates the interfacial property. Inserting source terms, momentum equation yields;

$$\frac{\partial}{\partial t}(\varepsilon\alpha_k\rho_k\mathbf{u}_k) + \nabla\cdot(\varepsilon\alpha_k\rho_k\mathbf{u}_k\mathbf{u}_k) =$$
$$-\nabla\cdot(\varepsilon\alpha_k p_k\mathbf{I}) + \nabla\cdot\left(\varepsilon\alpha_k\left(\mathcal{T}_k + \mathcal{T}_k^T\right)\right) - \left(1/V^T\right)\int_{A^I}\alpha_k\left(p_k\mathbf{I}_k\right)\cdot d\mathbf{A}^I \tag{27}$$
$$+\left(1/V^T\right)\int_{A^I}\alpha_k\mathcal{T}_k\cdot d\mathbf{A}^I + \varepsilon\alpha_k\rho_k\mathbf{g}_k + \varepsilon\mathbf{M}_k^\gamma + \varepsilon p_{ki}\nabla\alpha_k + \varepsilon\mathbf{M}_{ik} - \nabla\alpha_k\cdot\mathcal{T}_{ki}$$

Various terms in momentum equation can be modeled;

$$p_{ki} \cong p_k,\ \nabla\cdot\left(\varepsilon\alpha_k\left(\mathcal{T}_k + \mathcal{T}_k^T\right)\right) \cong \nabla\cdot\left(\varepsilon\alpha_k\nabla\left(\rho_k\nu_k\mathbf{u}_k\right)\right),\ \mathbf{M}_k^\gamma \cong \gamma_k\mathbf{u}_{ki} \tag{28}$$

$$\left(1/V^T\right)\int_{A^I}\alpha_k\left(p_k\mathbf{I}_k\right)\cdot d\mathbf{A}^I \cong K_w\left(\rho_k/2D_h\right)|\mathbf{u}_k|\mathbf{u}_k \tag{29}$$

$$\left(1/V^T\right)\int_{A^I}\alpha_k\mathcal{T}_k\cdot d\mathbf{A}^I \cong -\alpha_k C_{fk}\left(\rho_k/2D_h\right)|\mathbf{u}_k|\mathbf{u}_k \tag{30}$$

$$\mathbf{M}_{ik} \cong -C_{kk'}\left(\rho_k/2D_k\right)|\mathbf{u}_k - \mathbf{u}_{k'}|\left(\mathbf{u}_k - \mathbf{u}_{k'}\right),\ \nabla\alpha_k\cdot\mathcal{T}_{ki} \cong 0 \tag{31}$$

Then momentum equation is written;

$$(\partial/\partial t)(\varepsilon\alpha_k\rho_k\mathbf{u}_k) + \nabla\cdot(\varepsilon\alpha_k\rho_k\mathbf{u}_k\mathbf{u}_k) = -\varepsilon\alpha_k\nabla\cdot p_k + \nabla\cdot\left(\varepsilon\alpha_k\nabla\left(\rho_k\nu_k\mathbf{u}_k\right)\right)$$
$$-\varepsilon\alpha_k\left(\left(K_w + C_{fk}\right)\rho_k/2D_h\right)|\mathbf{u}_k|\mathbf{u}_k - \varepsilon C_{kk'}\left(\rho_k/2D_k\right)|\mathbf{u}_k - \mathbf{u}_{k'}|\left(\mathbf{u}_k - \mathbf{u}_{k'}\right)$$
$$+\varepsilon\alpha_k\rho_k\mathbf{g}_k + \varepsilon\gamma_k\mathbf{u}_{ki} \tag{32}$$

Kinetic energy equation is derived from momentum equation multiplying $\mathbf{u}_k$, and using mass equation;

$$(\partial/\partial t)\left(\varepsilon\rho_k\alpha_k\left(\mathbf{u}_k{}^2/2\right)\right) + \nabla\cdot\left(\varepsilon\rho_k\alpha_k\left(\mathbf{u}_k{}^2/2\right)\mathbf{u}_k\right) + \left(\mathbf{u}_k{}^2/2\right)\varepsilon\gamma_k =$$
$$+\mathbf{u}_k\nabla\cdot\left(\varepsilon\alpha_k\left(\mathcal{T}_k + \mathcal{T}_k^T\right)\right) - \mathbf{u}_k\left(1/V^T\right)\int_{A^I}\alpha_k\left(p_k\mathbf{I} - \mathcal{T}_k\right)\cdot d\mathbf{A}^I \tag{33}$$
$$-\mathbf{u}_k\nabla\cdot(\varepsilon\alpha_k p_k\mathbf{I}) + \varepsilon\mathbf{u}_k\alpha_k\rho_k\mathbf{g}_k + \varepsilon\mathbf{u}_k\left(\mathbf{M}_k^\Gamma + p_k\nabla\alpha_k + \mathbf{M}_{ik} - \nabla\alpha_k\cdot\mathcal{T}_{ki}\right)$$

Inserting source terms into total energy equation and subtracting kinetic energy one gets;

$$(\partial/\partial t)\left(\varepsilon\alpha_k\rho_k\left(e_k + \left(\mathbf{u}_k{}^2/2\right)\right)\right) - (\partial/\partial t)\left(\varepsilon\rho_k\alpha_k\left(\mathbf{u}_k{}^2/2\right)\right)$$
$$+\nabla\cdot\left(\varepsilon\alpha_k\rho_k\left(e_k + \left(\mathbf{u}_k{}^2/2\right)\right)\mathbf{u}_k\right) - \nabla\cdot\left(\varepsilon\rho_k\alpha_k\left(\mathbf{u}_k{}^2/2\right)\mathbf{u}_k\right) =$$
$$-\nabla\cdot\left(\varepsilon\alpha_k\left(\mathbf{q}_k + \mathbf{q}_k^T\right)\right) - \nabla\cdot\left(\varepsilon\alpha_k\left(p_k\mathbf{I} - \mathcal{T}_k\right)\cdot\mathbf{u}_k\right)$$
$$-\left(1/V^T\right)\int_{A^I}\alpha_k\left(\mathbf{q}_k - \mathbf{T}_k\cdot\mathbf{u}_k\right)\cdot d\mathbf{A}^I + \varepsilon\alpha_k\rho_k\mathbf{g}_k\cdot\mathbf{u}_k + \varepsilon\gamma_k\left(h_{ki} + \mathbf{u}_{ki}\cdot\mathbf{u}_k - \left(\mathbf{u}_k{}^2/2\right)\right)$$
$$+\varepsilon a_i q_{ki}'' - \varepsilon p_{ki}\frac{\partial\alpha_k}{\partial t} + \varepsilon\mathbf{u}_{ki}\cdot\mathbf{M}_{ik} - \varepsilon\nabla\alpha_k\cdot\mathcal{T}\cdot\mathbf{u}_{ki} + \varepsilon W_{ki}^T + \mathbf{u}_k\nabla\cdot(\varepsilon\alpha_k p_k\mathbf{I})$$
$$-\mathbf{u}_k\nabla\cdot\left(\varepsilon\alpha_k\left(\mathcal{T}_k + \mathcal{T}_k^T\right)\right) + \left(1/V^T\right)\mathbf{u}_k\int_{A^I}\alpha_k\left(p_k\mathbf{I} - \mathcal{T}_k\right)\cdot d\mathbf{A}^I - \varepsilon\mathbf{u}_k\alpha_k\rho_k\mathbf{g}_k$$
$$-\varepsilon\mathbf{u}_k\left(\mathbf{M}_k^\Gamma + p_{ki}\nabla\alpha_k + \mathbf{M}_{ik} - \nabla\alpha_k\cdot\mathcal{T}_{ki}\right) + \left(\mathbf{u}_k{}^2/2\right)\varepsilon\gamma_k \tag{34}$$

Simplifying yields the internal energy equation;

$$(\partial/\partial t)(\varepsilon\alpha_k\rho_k e_k) + \nabla\cdot(\varepsilon\alpha_k\rho_k e_k\mathbf{u}_k) = -\varepsilon\alpha_k p_k\nabla\cdot\mathbf{u}_k - \varepsilon\mathbf{u}_k p_{ki}\nabla\alpha_k$$
$$-\varepsilon p_{ki}(\partial/\partial t)\alpha_k + \varepsilon\gamma_k h_{ki} + \varepsilon a_i q_{ki}'' - \overbrace{\nabla\cdot\left(\varepsilon\alpha_k\left(\mathbf{q}_k + \mathbf{q}_k^T + \mathcal{T}_k^T\cdot\mathbf{u}_k\right)\right)}^{IA} \tag{35}$$
$$+\varepsilon\overbrace{\alpha_k\left(\mathcal{T}_k + \mathcal{T}_k^T\right)\nabla\mathbf{u}_k}^{IB} - \varepsilon\overbrace{\nabla\alpha_k\cdot\mathcal{T}_{ki}\cdot\left(\mathbf{u}_{ki} - \mathbf{u}_k\right)}^{IC} + \varepsilon\overbrace{\mathbf{M}_{ik}\left(\mathbf{u}_{ki} - \mathbf{u}_k\right)}^{ID} + \varepsilon\overbrace{W_{ki}^T}^{IE}$$
$$\overbrace{-\left(1/V^T\right)\int_{A^I}\alpha_k\left(\mathbf{q}_k + \mathbf{T}_k\cdot\mathbf{u}_k\right)\cdot d\mathbf{A}^I}^{IF} + \overbrace{\left(1/V^T\right)\mathbf{u}_k\int_{A^I}\alpha_k\left(p_k\mathbf{I} - \mathcal{T}_k\right)\cdot d\mathbf{A}^I}^{IG}$$

Modeling various terms in energy equation;

$$p_k = p_{ki} \tag{36}$$

$$IA = \nabla\cdot\left(\varepsilon\alpha_k\left(\mathbf{q}_k + \mathbf{q}_k^T + \mathcal{T}_k^T\cdot\mathbf{u}_k\right)\right) = -\nabla\cdot\left(\varepsilon\alpha_k\mu_k^{eT}\nabla e_k\right) \tag{37}$$

$$IE = -\left(1/V^T\right)\int_{A^I}\alpha_k\left(\mathbf{q}_k + \mathbf{T}_k\cdot\mathbf{u}_k\right)\cdot d\mathbf{A}^I = q_k^w/D_h \tag{38}$$

IB, *IC*, *ID*, *IF* and *IG* ; neglected

Then, internal energy equation is written;

$$(\partial/\partial t)(\varepsilon\alpha_k\rho_k e_k) + \nabla\cdot(\varepsilon\alpha_k\rho_k e_k\mathbf{u}_k) - \nabla\cdot\left(\varepsilon^V\alpha_k\mu_k^{eT}\nabla e_k\right) =$$
$$-\varepsilon\alpha_k p\nabla\cdot(\alpha_k\mathbf{u}_k) - \varepsilon p(\partial/\partial t)\alpha_k + \varepsilon\gamma_k h_{ki} + \varepsilon a_i q_{ki}'' - q_k^w/D_h \tag{39}$$

Mass and energy jump conditions are written;

$$\sum\gamma_k = 0$$
$$\sum\left(\gamma_k h_{ki} + a_i q_{ki}''\right) = 0 \tag{40}$$

4. Conclusions

Governing equations for the multi-fluid flow over a porous body has been successfully derived using the time-volume averaging procedure. Internal energy balance equation is also derived. These equations may be regarded as the basis of the balance equations for a computer program such as SPACE. The porous body approach will give us a systematic way to handle the system geometric data for the multi-dimensional applications via the CAD tools as well as the mesh generators.

References

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