Improvement of SPACE Momentum Equation to Mitigate Non-physical Pressure Solution

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1. Introduction

Non-physical pressure solutions are often numerically induced, when two-fluid model is applied to liquid-fill problems for initially empty vertical pipe, especially at low pressure condition: One of the reasons for the numerical error is the "water-packing" phenomenon. In other words, when the liquid level crosses the cell interfaces of the finite volume mesh, the lower cell is sometimes filled with too large amount of liquid to accommodate without pressure excursion. Therefore, most of two-fluid system codes adopt the "water-packing mitigation" scheme, which is based on the numerical technique where the spurious pressure spikes are suppressed by not only providing artificially a small influence coefficient of the excess mass flow on the cell pressure, but also forcing up-flow at the level-crossing face with a finite value of velocity. On the other hand, another reason to cause the spurious pressure solution has been found by Murao[1]. This is called "momentum imbalance" of the two-fluid models using phasic momentum equation. It is related to the numerical solution process in which the physically meaningless liquid velocity calculated for the face above the liquid level is converted to the real liquid velocity at the time of level-crossing. In order to eliminate the momentum imbalance, Murao has invented an ad-hoc function multiplier for the convection, pressure gradient, and gravity terms of each phasic momentum equation. MARS code uses an extensive form of sum momentum equation describing the conservation of the mixture momentum of two-fluids, and phase intensive form of difference momentum equation. In the previous version of SPACE, phase intensive form of momentum equation is used with Murao type ad-hoc function multiplier for the pressure gradient and gravity terms. In this study, some more detailed discussion is made on the momentum imbalance induced by the phase intensive momentum equation, and the improved sum-difference type SPACE momentum equations are described. Finally, assessment of the typical liquid-fill problem, Prypor pipe test[2], is performed, and the results are compared with those of the Murao's two-fluid model and sum-difference model of MARS.

2. Discussion on the momentum imbalance

2. 1 Original phase intensive momentum equation

Momentum equations of the two-fluid model can be used in a variety of different types, as mentioned before. Among them, the phasic momentum equation is used for the reason that the momentum matrix, constructed in a face neglecting the neighbor face coupling, has no singularity, because the temporal term of momentum equation is written only by the phasic velocity itself. The phase intensive form of momentum equation can be written as follows.

$$\frac{\partial \mathbf{U}_g}{\partial t} + \mathbf{U}_g \cdot \nabla \mathbf{U}_g = -\frac{1}{\rho_g} \nabla P + \mathbf{g} + \mathbf{S}_g \quad (1)$$

$$\frac{\partial \mathbf{U}_l}{\partial t} + \mathbf{U}_l \cdot \nabla \mathbf{U}_l = -\frac{1}{\rho_l} \nabla P + \mathbf{g} + \mathbf{S}_l \quad (2)$$

2. 2 Modified phase intensive momentum equation

The intensive form of momentum equation is beneficial to avoid matrix singularity, but it is pointed out by Murao that the "momentum imbalance" can take place at the time of level-crossing in liquid-fill problems. At the faces above liquid level, the pressure gradient is governed by the vapor momentum equation, and it is balanced with the vapor head, i.e., the multiplication of vapor density and gravity. On the other hand, the liquid momentum equation is not satisfied with the small gravity head, but it requires relatively larger liquid head instead of vapor head. Therefore, the liquid velocity increases in the downward direction, until the convection term becomes balanced with the required liquid head. Consequently, at the time of liquid level-crossing, the meaningless downward liquid velocity is converted in a very short time, to the real liquid velocity. In this process, the momentum imbalance can take place between the convection, pressure gradient and gravity terms of phasic momentum equation. In order to eliminate the momentum imbalance, Murao introduces the ad-hoc function multiplier for the momentum equation. The modified momentum equation proposed by Murao is as follows.

$$\begin{aligned} &\frac{\partial \mathbf{U}_g}{\partial t} + \frac{\alpha_g}{\max(\alpha_g, 10^{-8})} \mathbf{U}_g \cdot \nabla \mathbf{U}_g \\ &= -\frac{\alpha_g}{\max(\alpha_g, 10^{-8})} \frac{1}{\rho_g} \nabla P + \frac{\alpha_g}{\max(\alpha_g, 10^{-8})} \mathbf{g} + \mathbf{S}_g \end{aligned} \quad (3)$$

$$\frac{\partial \mathbf{U}_l}{\partial t}+\frac{\alpha_l}{\max(\alpha_l,10^{-8})}\cdot\mathbf{U}_l\nabla\mathbf{U}_l$$
$$=-\frac{\alpha_l}{\max(\alpha_l,10^{-8})}\frac{1}{\rho_l}\nabla P+\frac{\alpha_l}{\max(\alpha_l,10^{-8})}\mathbf{g}+\mathbf{S}_l \quad (4)$$

2.3 Sum-difference type momentum equation

In the previous version of the SPACE code, phase intensive momentum equations are used with Murao type ad-hoc function multipliers for the pressure gradient and gravity terms. One thing different is that the phasic volume fractions used for the multiplication factors are chosen to be the donor cell values instead of the face ones. The reason is that if there is inconsistency between the ad-hoc function multiplier for the phasic momentum equation and donor volume fraction used for the mass or energy equation, it also induces a kind of "momentum imbalance" different from those of Murao. In this model, however, the required pressure gradient can abruptly change from the vapor head to the liquid head, when the liquid-level crosses the face. Therefore, sum momentum equation is introduced to the latest version of SPACE. The sum momentum equation makes the total pressure gradient to be balanced with the mixture head, regardless of absence of one or two phases. In the difference equations, the same ad-hoc function multipliers based on the donor volume fraction are used as in the aforementioned phasic momentum equation.

3. Test results

In this section, the various types of momentum equations are applied to the assessment of the low pressure liquid-fill problem, Pryor pipe test. Fig.1 compares the calculation results from the original phase intensive form of momentum equation and those from the modified equation proposed by Murao. The original phase intensive momentum equation results in step change of the pressure at every time of level crossing. In the modified momentum equation by Murao, the step change of pressure is significantly mitigated. But a certain level of pressure jump remains, since there is still momentum imbalance between the pressure gradient demanded by the phasic momentum equation and the actual pressure built-up by the mass or energy flow. Fig. 2 shows the comparison of the results of the SPACE and MARS sum-difference momentum equations. The sum equation results in the overall pressure gradient being balanced with the multiplication of mixture density at faces. MARS code result shows a good linear variation of pressure with little pressure spikes or jumps at the time of level crossing. But the SPACE code results show a better agreement on the analytic solution.

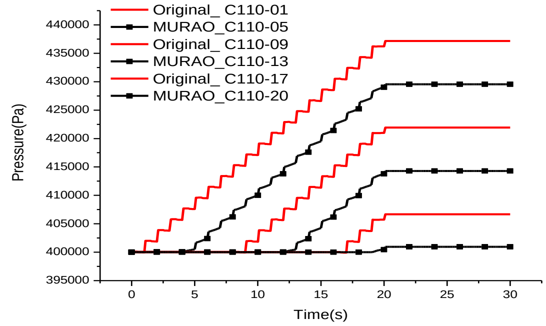

Fig. 1 Comparison of cell pressures (original equation vs. the equation modified by Murao)

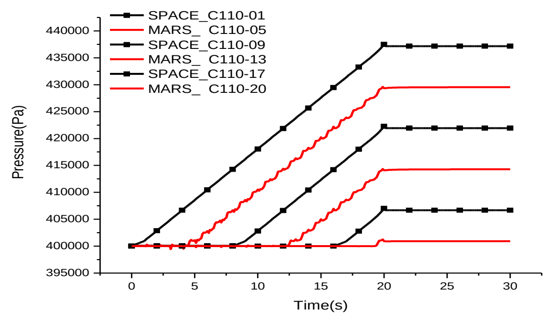

Fig. 2 Comparison of cell pressures (SPACE vs. MARS)

4. Conclusion

Various types of momentum equations are assessed for the liquid-fill problem at a vertical pipe. As a result, the sum-difference equations modified with the Murao type multiplication factor applied to the convection, pressure gradient terms of the difference equation, show the best agreement on the analytic solution. The modified sum-difference momentum equations are incorporated into the latest version of the SPACE code.

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