Equivalent Young's Modulus of Perforated Shell with Triangular Penetration Pattern

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1. Introduction

The analysis of a plate or shell perforated with a large number of holes, by finite element method for instance, was a very costly and time-consuming technique which solves only one particular problem. But it is possible to model the perforated plate or shell and to analyze it and it is no more time-consuming theses days due to the rapid development of the computer and software. However, if a perforated plate or shell is submerged in fluid it is almost impossible to model and analyze it as is and the fluid at the same time, which is needed to investigate the effect of the fluid-structure interaction. The simplest way to avoid time-consuming and costly analysis of perforated plate or shell submerged in fluid is to replace the perforated plate or shell by an equivalent solid one considering weakening effect of holes.

Many authors have proposed experimental or theoretical method to solve this problem for the plate. Slot and O'Donnell [1] determined the effective elastic constants for the thick perforated plates by equating strains in the equivalent solid material to the average strains in the perforated material. O'Donnell [2] also presented those of thin perforated plates. These results are implemented in Article A-8000 of Appendix A to the ASME code Section III, which contains a method of analysis for flat perforated plates when subjected to directly applied loads or loadings resulting from structural interaction with adjacent members.

Unfortunately the effective elastic constants for the perforated shell are not found in any references. Therefore in this study the modal characteristics of the perforated shell are investigated and the equivalent material properties of perforated shell are suggested by performing several finite element analyses with respect to the ligament efficiencies.

2. Analysis

Considering a cylindrical perforated shell with a triangular penetration pattern as shown in Fig. 1, where the mean radius of the shell and pitch of the hole are 99.2 mm and 36 mm, respectively, the modal characteristics and the equivalent elastic constants are investigated as a typical case.

Frequencies of solid shell and perforated shell of ligament efficiency $\eta = 0.05$ with triangular penetration pattern are shown in Fig. 2. The inclusion of holes decreases the frequency significantly and this is more significant as the modal number decreases.

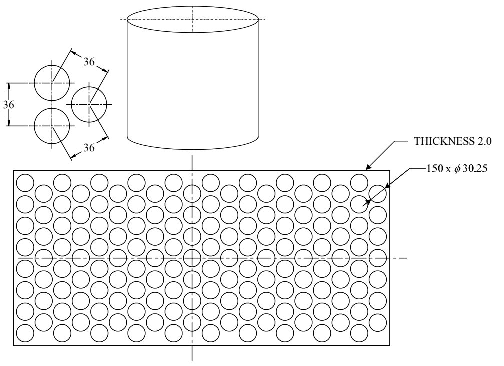

Fig. 1. Perforated shell with triangular penetration pattern

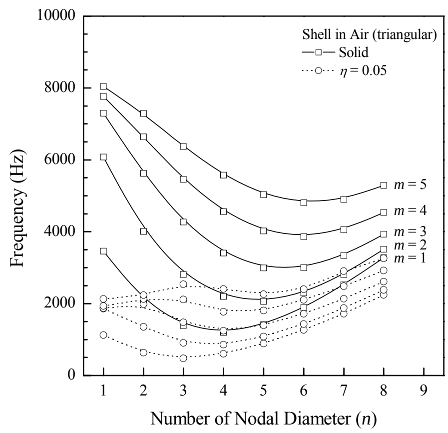

Fig. 2. Frequency comparisons between solid and perforated shells in air

As mentioned earlier, the equivalent elastic constants of perforated shell are not found anywhere. Therefore it is necessary to define the equivalent elastic constants such that the modal characteristics of the perforated shell with original properties be the same with those of the solid shell with modified equivalent properties. The best way to find the equivalent constant is as follows:

(1) Develop finite element model of solid shell and perforated shells with various ligament efficiencies (Fig. 3).
(2) Perform the modal analyses of perforated and solid shell with original properties.
(3) Compare the frequencies and find the ratio of frequencies of perforated shell to those of solid shell.
(4) Find the multipliers of Young's modulus of solid shell to match frequencies of perforated shell with original properties using the relations between frequency and Young's modulus.

(5) Effective elastic constant for each ligament efficiency is averaged for all modes.

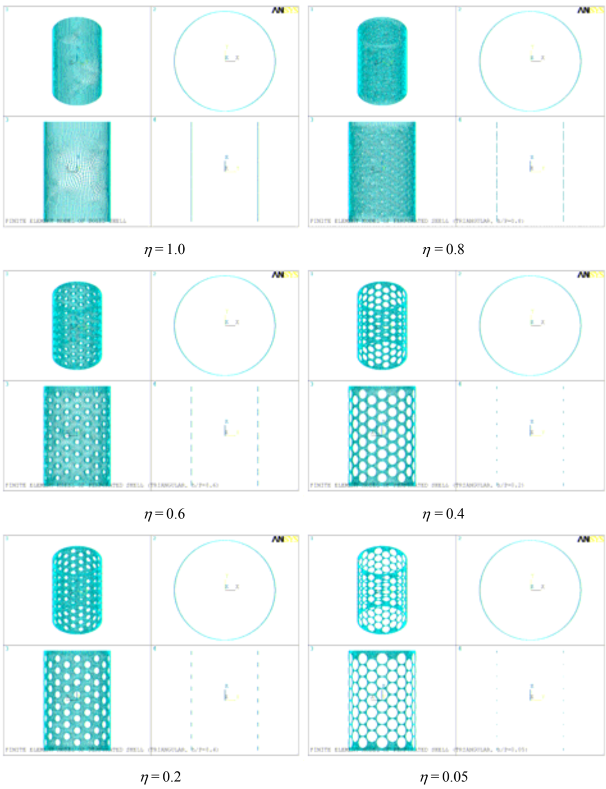

Fig. 3. Finite element models of solid and perforated shells with a triangular penetration pattern

The natural frequencies are summarized in Fig. 4 and their normalized values of perforated shell with respect to the solid shell are calculated. Because the elastic constant is proportional to the square of the frequency, the normalized frequencies are squared and these are the effective elastic constants with which the same modal frequencies as the perforated shell are obtained for the solid shell. Because the effective values are dependent on the mode numbers, their average values are used in Fig. 5, which is the final effective elastic constants proposed for modal characteristics of the perforated shell with a triangular penetration pattern as;

$$\frac{E^*}{E} = 0.1610 + 1.7421\eta - 2.0365\eta^2 + 2.2733\eta^3 - 1.1471\eta^4$$

Because the effective elastic constants are much changed depending on the mode numbers it is not proper to define one effective value for each ligament efficiency. For example, the effective elastic constant varies from the minimum 0.062 to the maximum 0.468 with the average of 0.242 for η = 0.05. The frequency errors are expected for the case of using average equivalent elastic constants defined above. Therefore the equivalent elastic constant defined above should be limited only for $0.5 \le \eta \le 0.8$ for the frequency errors of the perforated shell to be within 10 %.

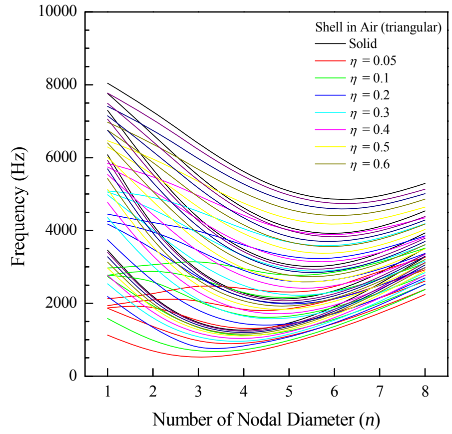

Fig. 4. Natural frequencies of shell in air for solid shell and perforated shells with a triangular penetration pattern

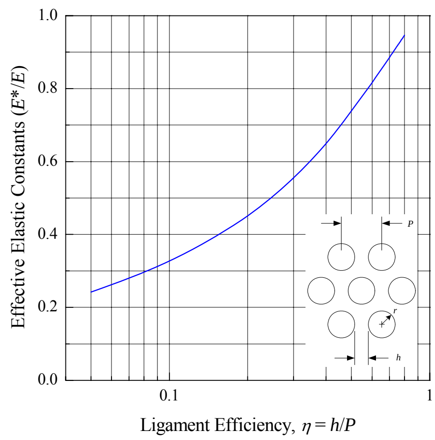

Fig. 5. Effective elastic constants of perforated shell with a triangular penetration pattern

3. Conclusions

For the perforated shell submerged in fluid, it is almost impossible to develop a finite element model due to the necessity of the fine meshing of the shell and the fluid at the same time. This necessitates the use of solid shell with equivalent material properties. Therefore in this study the equivalent Young's modulus of perforated shell are suggested by performing several finite element analyses with respect to the ligament efficiencies.

REFERENCES

[1] Slot, T., and O'Donnell, W.J., 1971, Effective Elastic Constants for Thick Perforated Plates with Triangular and Square Penetration Patterns, *Journal of Engineering for Industry*, 93(4), pp.935-942.

[2] O'Donnell, W.J., 1973, Effective Elastic Constants for the Bending of Thin Perforated Plates with Triangular and Square Penetration Patterns, *Journal of Engineering for Industry*, 95, pp.121-128.