Development of a 2-D Simplified P3 FEM Solver for Arbitrary Geometry Applications

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1. Introduction

In the calculation of power distributions and multiplication factors in a nuclear reactor, the Finite Difference Method (FDM) and the nodal methods are primarily used. These methods are, however, limited to particular geometries and lack general application involving arbitrary geometries. The Finite Element Method (FEM) can be employed for arbitrary geometry application and there are numerous FEM codes to solve the neutron diffusion equation or the Sn transport equation. The diffusion based FEM codes [1,2] have the drawback of inferior accuracy while the Sn based ones require a considerable computing time. This work here is to seek a compromise between these two by employing the simplified P3 (SP3) method for arbitrary geometry applications. Sufficient accuracy with affordable computing time and resources would be achieved with this choice of approximate transport solution when compared to full FEM based Pn [3] or Sn solutions. For now only 2-D solver is considered.

2. Modified SP3 Equation and Galerkin Method

2.1 SP3 Equation and Vacuum B.C.

The original SP3 equation [4] involves a nonsymmetric formulation. This can be converted into a symmetric form easily which would give the benefit of allowing the use of the basic Krylov subspace method such as the conjugate gradient method in the solution of the resulting linear system. In this regard, the following form of the SP3 equations with standard notations are used in this research.

$$\begin{bmatrix} -D_{0g}\nabla^2+\Sigma_{rg} & -2\Sigma_{rg} \\ -2\Sigma_{rg} & -3D_{2g}\nabla^2+4\Sigma_{rg}+5\Sigma_{tg} \end{bmatrix}\begin{bmatrix} \hat{\Phi}_g \\ \Phi_{2g} \end{bmatrix}=\begin{bmatrix} q_{0g} \\ -2q_{0g} \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} J_{0g} \\ 3J_{2g} \end{bmatrix}=\begin{bmatrix} 1/2 & -3/8 \\ -3/8 & 21/8 \end{bmatrix}\begin{bmatrix} \hat{\Phi}_{sg} \\ \Phi_{2g} \end{bmatrix} \quad (2)$$

2.2 Symmetric & Positive Definite Matrix with WRM

Applying the Galerkin method to the above SP3 equation yields the following block 3x3 matrix consisting of 2x2 blocks.

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}\begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}=\begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} \quad (3)$$

where

$$A_{ij}(1,1)=D_{0g}\iint_{\Omega}\frac{\partial\psi_i}{\partial x}\frac{\partial\psi_j}{\partial x}+\frac{\partial\psi_i}{\partial y}\frac{\partial\psi_j}{\partial y}d\Omega +\Sigma_{rg}\iint_{\Omega}\psi_i\psi_j d\Omega+\int_{\partial\Omega}D_{0g}\frac{P_+}{O_+}\psi_i\psi_j dl$$

$$A_{ij}(1,2)=-2\Sigma_{rg}\iint_{\Omega}\psi_i\psi_j d\Omega+\int_{\partial\Omega}D_{0g}\frac{Q_+}{O_+}\psi_i\psi_j dl$$

$$A_{ij}(2,1)=-2\Sigma_{rg}\iint_{\Omega}\psi_i\psi_j d\Omega+\int_{\partial\Omega}D_{2g}3\frac{P_2}{O_2}\psi_i\psi_j dl$$

$$A_{ij}(2,2)=3D_{2g}\iint_{\Omega}\frac{\partial\psi_i}{\partial x}\frac{\partial\psi_j}{\partial x}+\frac{\partial\psi_i}{\partial y}\frac{\partial\psi_j}{\partial y}d\Omega +\left(4\Sigma_{rg}+5\Sigma_{tg}\right)\iint_{\Omega}\psi_i\psi_j d\Omega+\int_{\partial\Omega}D_{2g}3\frac{Q_2}{O_2}\psi_i\psi_j dl$$

$$B_i(1)=\hat{\Phi}_{gi},\ B_i(2)=\Phi_{2gi}$$

$$C_i(1)=\iint_{\Omega}q_{0g}\psi_i d\Omega+\int_{\partial\Omega}D_{0g}\frac{R_+}{O_+}\psi_i dl$$

$$C_i(2)=-2\iint_{\Omega}q_{0g}\psi_i d\Omega+\int_{\partial\Omega}D_{2g}3\frac{R_2}{O_2}\psi_i dl$$

Here, Ψ_i denotes the basis function of the i-th corner. O,P,Q,R are the coefficients in the general coupled boundary condition equation. The current relation is given below:

$$\begin{bmatrix} J_{0g} \\ 3J_{2g} \end{bmatrix}=\begin{bmatrix} -D_{0g}\frac{\partial\hat{\Phi}_g}{\partial n} \\ -3D_{2g}\frac{\partial\Phi_{2g}}{\partial n} \end{bmatrix}=\begin{bmatrix} D_{0g}\frac{P_+}{O_+} & D_{0g}\frac{Q_+}{O_+} \\ 3D_{2g}\frac{P_2}{O_2} & 3D_{2g}\frac{Q_2}{O_2} \end{bmatrix}\begin{bmatrix} \hat{\Phi}_g \\ \Phi_{2g} \end{bmatrix}+\begin{bmatrix} -D_{0g}\frac{R_+}{O_+} \\ -3D_{2g}\frac{R_2}{O_2} \end{bmatrix} \quad (4)$$

The mesh generation for the FEM calculation can be done by various open mesh generation programs. Here the GMSH utility is used. Both the linear and quadratic basis function options are employed. The resulting linear system is solved by the preconditioned Krylov subspace method.

3. Verification

For the verification of the newly developed SP3 FEM solver, the Takeda fast reactor benchmark [5], the IAEA 2D benchmark [1] and etc are solved with various methods.

3.1 Takeda Benchmark

This fast reactor benchmark problem shown in Figure 1 is a fast reactor problem in which the transport effect is significant. [5]

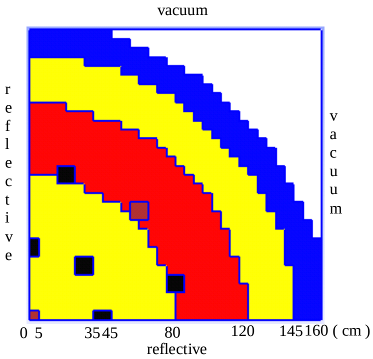

Fig. 1. Takeda 2D Model 3 Domain

This problem was solved with the following meshes and basis functions.

Table I: FEM Input for Takeda 2D Model 3,

Basis Ftn.	Nodes	Elements	Avg. Area
Linear	6427	12522	2.044 cm^2
Quadratic	6369	3094	8.274 cm^2

The multiplication factors obtained with various codes and methods are shown below.

Table II: k_{eff} Result for Takeda 2D Model 3

McCARD	DeCART	SP3 FDM	SP3 FEM	
0.90363 (0)	0.90363 (0)	0.90270 (-93)	Linear	0.90190 (-173)
			Quadratic	0.90236 (-127)

3.2 IAEA 2D Hex Core Benchmark with Rods Inserted

For the following hex core IAEA problem [1], the same cases yield the results shown in Tables IV and V and Fig.3

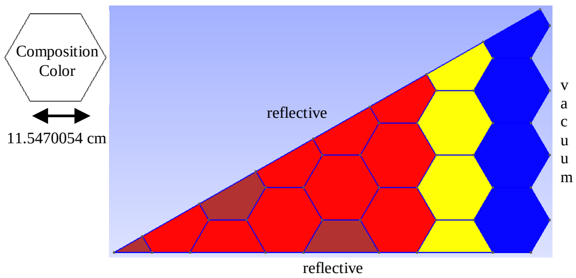

Fig. 2. IAEA 2D Benchmark Domain

Table III: FEM Input for IAEA 2D Benchmark

Basis Ftn.	Nodes	Elements	Avg. Area
Linear	4655	8985	0.543 cm^2
Quadratic	4543	2186	2.232 cm^2

Table IV: k_{eff} Result for IAEA 2D Benchmark

McCARD	DeCART	SP3 FDM	SP3 FEM	
1.00622 (0)	1.00675 (+53)	1.00624 (+2)	Linear	1.00633 (+11)
			Quadratic	1.00625 (+3)

Table V : Power RMS Error for IAEA 2D Benchmark

	Linear FEM	Quadratic FEM
RMS Error (%)	0.252	0.178

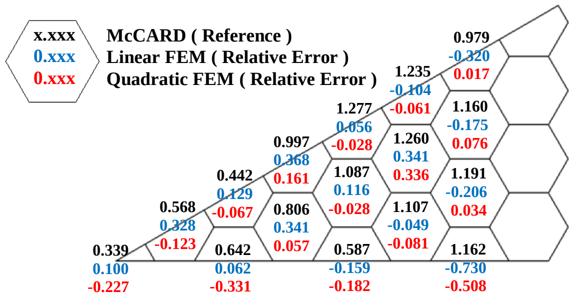

Fig. 3.Power Result for IAEA 2D Benchmark

3.3 Monju 2D Benchmark

This is a representative rotational problem [1]. Results are given in Table VII

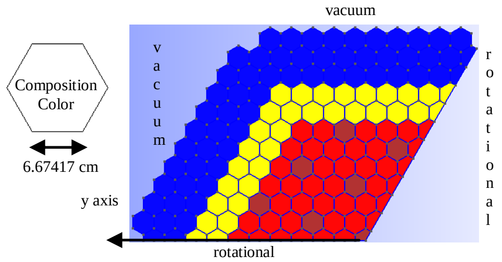

Fig. 4. Monju 2D Benchmark Domain

Table VI. FEM Input for MONJU 2D Benchmark

Basis Ftn.	Nodes	Elements	Avg. Area
Linear	6390	12448	3.634 cm^2
Quadratic	6637	3230	14.005 cm^2

Table VII. k_{eff} Result for MONJU 2D Benchmark,

SP3 FEM	
Linear	1.15831
Quadratic	1.15848

4. Conclusions

By solving various problems, it was demonstrated that the 2-D SP3 FEM solver gives reasonable accuracy compared with the other higher order transport methods such as Monte Carlo (McCARD) and MOC(DeCART). With the good results shown for hexagonal or square geometries, it is expected that the accuracy of FEM would be retained even for an arbitrary geometry. Extension to 3-D applications is under way.

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