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# Calculation of The Characteristic Limits for a Surface Contamination Ratemeter

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## 1. Introduction

Characteristic limits such as decision threshold, detection limit and confidence limits give an answer to the following questions: (1) Is the measurement procedure used suitable for the intended measurement purpose? (2) Is the number of event collected be attributed to the radioactive element? (3) Which range of true values may be reasonably attributed to the measurand given the measured results? ISO provided the theoretical framework and rules for the calculation of characteristics limits for ionizing radiation[1]. This paper gives an example of applications of ISO for a contamination surface ratemeter.

## 2. Methods and Results

In this section equations for the calculation of characteristics limits for a surface contamination ratemeter are provided, and a numerical example is given.

### *2.1 Symbols Employed*

For the purpose of this paper, the following symbols are used:

| | |
|---|---|
| $y, u(y)$ | measured value, and standard uncertainty associated with y, respectively |
| $\tilde{y}, \tilde{u}(\tilde{y})$ | true value, and standard uncertainty associated with $\tilde{y}$, respectively |
| $\hat{y}, u(\hat{y})$ | best estimate, and standard uncertainty associated with $\hat{y}$, respectively |
| $y^*$ | decision threshold |
| $y\#$ | detection limit |
| $\alpha, \beta$ | probability of the error of the first and second kind, respectively |
| $1-\gamma$ | probability for the confidence interval of the measurand |
| $y^{\triangleleft}, y^{\triangleright}$ | lower and upper limit of the confidence interval, respectively, of the measurand |
| $k_p, k_q$ | quantiles of the standardized normal distribution for the probability p and q, respectively |
| $\Phi$ | distribution function of the standardized normal distribution |
| $\tau_g, \tau_0$ | relaxation time constant of a ratemeter used for the measurement of the gross effect and of the background effect, respectively |
| $r_g, r_0$ | estimate of the gross count rate and of background count rate, respectively, in cps |
| $\varepsilon, A$ | detection efficiency in $Bq^{-1}.s^{-1}$, and detector probe area in $cm^2$, respectively |

### *2.2 Mathematical Model*

Generally, the mathematical model for the calculation of the measured value, in becquerels per square centimeter ($Bq.cm^{-2}$), by use of a surface contamination meter is given by:

$$y = (r_g - r_0) \cdot w \tag{1}$$

with

$$w = \frac{1}{\varepsilon \cdot A} \tag{2}$$

The standard uncertainty u(y) associated with y is given by:

$$u^2(y) = w^2 \cdot \left( \frac{r_g}{2\tau_g} + \frac{r_0}{2\tau_0} \right) + y^2 u_{rel}^2(w) \tag{3}$$

with

$$u_{rel}^2(w) = \frac{u^2(\varepsilon)}{\varepsilon^2} + \frac{u^2(A)}{A^2} \tag{4}$$

Replacing y by $\tilde{y}$ and eliminating $r_g = \tilde{y}/w + r_0$, standard uncertainty as a function of the true value $\tilde{u}(\tilde{y})$ is given by:

$$\tilde{u}^2(\tilde{y}) = w^2 \cdot \left[ \frac{\tilde{y}}{2\tau_g \cdot w} + r_0 \cdot \left( \frac{1}{2\tau_g} + \frac{1}{2\tau_0} \right) \right] + \tilde{y}^2 u_{rel}^2(w) \tag{5}$$

### *2.3 Decision Threshold*

The decision threshold *y** is a characteristic limit, which, when exceeded by a result y of a measurement helps one to decide that radionuclide is present in the sample. If $y \le y^*$ one decides that radionuclide is not found in the sample. If this decision rule is observed, a wrong decision on favor of the presence of radionuclide occurs with a probability not greater than $\alpha$. The decision threshold is given by[1,2]:

$$y^* = k_{1-\alpha} \tilde{u}(0) \tag{6}$$

Substituting $\tilde{y} = 0$ into Equation (5) then one obtains the decision threshold:

$$y^* = k_{1-\alpha} \cdot w \cdot \sqrt{r_0 \cdot \left( \frac{1}{2\tau_g} + \frac{1}{2\tau_0} \right)} \tag{7}$$

### *2.4 Detection Limit*

The detection limit $y^{\#}$ is the smallest true value of the measurand detectable with the measuring method. The detection limit $y^{\#}$ is sufficiently larger than the decision threshold $y^*$ such that the probability of y< $y^{\#}$ equals the probability β of the error of the second kind in the case of $\tilde{y} = y^{\#}$. The detection limit is given by[1,2] :

$$y^{\#} = y^{*} + k_{1-\beta}\tilde{u}(y^{\#}) \quad (8)$$

Substituting $\tilde{y} = y^{\#}$ into Equation (5) then one obtains the detection limit:

$$y^{\#} = y^{*} + k_{1-\beta} \cdot \sqrt{w^2 \cdot \left[ \frac{y^{\#}}{2\tau_g \cdot w} + r_0 \cdot \left( \frac{1}{2\tau_g} + \frac{1}{2\tau_0} \right) \right] + y^{\#2} \cdot u_{rel}^2(w)} \quad (9)$$

### *2.5 Confidence Limits*

For a result y of a measurement which exceeds the decision threshold y*, the confidence interval includes the true value of the measurand with the given probability (1-γ). If y/u(y)<4, confidence limits are given by[1,2] :

$$y^{\triangleleft} = y - k_p \cdot u(y) \quad with \quad p = \omega \cdot (1 - \gamma/2) \quad (10)$$

$$y^{\triangleright} = y - k_q \cdot u(y) \quad with \quad q = 1 - (\omega \cdot \gamma/2) \quad (11)$$

with

$$\omega = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y/u(y)} exp\left(-z^2/2\right) dz = \Phi(y/u(y)) \quad (12)$$

But if y/u(y) ≥4 confidence limits are calculated as in conventional statistics as follows:

$$y^{\triangleleft} = y - k_{1-\gamma/2} u(y) \quad and \quad y^{\triangleright} = y + k_{1-\gamma/2} u(y) \quad (13)$$

### *2.6 Assessment of a measured result*

If y/u(y) <4, best estimate $\hat{y}$ is given by[1,2]:

$$\hat{y} = y + \frac{u(y) \cdot exp\left\{-y^2/[2u^2(y)]\right\}}{\omega \cdot \sqrt{2\pi}} \quad (14)$$

with the u( $\hat{y}$ ) associated $\hat{y}$ :

$$u(\hat{y}) = \sqrt{u^2(y) - (\hat{y} - y) \cdot \hat{y}} \quad (15)$$

But if y/u(y)≥4, the approximation $\hat{y} = y$ and $u(\hat{y}) = u(y)$ is valid[1,2].

### *2.6 Numerical Example*

As a numerical example, the calculation of the characteristic limits is described for a surface contamination meter. The probabilities α=β=γ=0.05 were chosen. This yields $k_{1-\alpha}$ =$k_{1-\beta}$ =1.645 and $k_{1-\gamma/2}$ =1.96. The decision threshold $y^*$ can be calculated using Equation (7) and data given in Table I. With α=0.05 and $k_{1-\alpha}$ =1.645 we obtain the result for the decision threshold of 0.0801 $s^{-1}$ or 0.0054 $Bq.cm^{-2}$. If the readings is >(0.0801+0.02)=0.1001$s^{-1}$, a contamination has been detected at error of the first kind of α=5 %. Activity per unit area, in becquerels per square centimeter, $y$ is calculated to be 0.0162 $Bq.cm^{-2}$ with u(y)=0.0079 $Bq.cm^{-2}$. Since measured value exceeds the decision threshold, a contamination is observed. The detection limit $y^{\#}$ can be calculated using Equation (9) and data given in Table I. With α or β =0.05 and $k_{1-\alpha}$ =$k_{1-\beta}$ =1.645, we obtain the result for the detection limit of 0.0195 $Bq.cm^{-2}$. Since detection limit is lower than the guideline value of 0.4 $Bq.cm^{-2}$, the measurement method is suitable for the purpose of the measurement. y/u(y) ratio is 2.06. Therefore, the confidence limit are calculated to be $y^{\triangleleft}$ =0.0028 $Bq.cm^{-2}$ and $y^{\triangleright}$ =0.0317 $Bq.cm^{-2}$ from Equations (10) and (11). Also, the best estimate $\hat{y}$ and its standard uncertainty $u(\hat{y})$ are calculated to be 0.0166 $Bq.cm^{-2}$ and 0.0075 $Bq.cm^{-2}$.

Table I: Data for example of rate-meter measurement

| Quantity | value | Standard uncertainty | Unit | Type of uncertainty |
|---|---|---|---|---|
| $r_g$ | 0.2 | 0.0816 | $s^{-1}$ | A |
| $\tau_g$ | 15 | - | s | - |
| $r_0$ | 0.02 | 0.0258 | $s^{-1}$ | A |
| $\tau_0$ | 15 | - | s | - |
| ε | 0.089 | $8.72x10^{-3}$ | $Bq^{-1}s^{-1}$ | B |
| A | 125 | 0 | $cm^2$ | B |
| Guideline value | 0.4 | - | $Bq.cm^{-2}$ | - |

## 3. Conclusions

The working expressions of characteristics limits for a surface contamination meters are provided. The expressions can be useful for solving three questions asked in the introduction

## REFERENCES

[1] ISO, International Standard Organization, ISO 11929, Determination of The Characteristic Limits(Decision Threshold, Detection Limit and Limits of the Confidence Interval) for Measurements of Ionizing Radiation-Fundamentals and General Applications, 2010.

[2] K. Weise, K. Hubel, E. Rose, M. Schlager, D. Schrammel, M. Taschner, and R. Michel, Bayesian decision Threshold, Detection Limit and Confidence Limits in Ionizing-radiation Measurement, Radiation Protection Dosimetry, Vol.121, No. 1, pp. 52-63, 2006.