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Performance Evaluation of Modified Single Parameter Hecht Equation

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1. Introduction

The semiconductor detectors have been widely used for radiation detection and medical imaging because of their outstanding features such as excellent energy resolution and room temperature operation. However, the performance of these detectors is, in fact, limited by the charge transport properties of the semiconductor, especially the electron and hole mobility-lifetime products (i.e., $(\mu\tau)_e$ and $(\mu\tau)_h$) [1]. Hence, the analysis on the mobility-lifetime products is very important for accurately evaluating the characteristics of the semiconductor detectors.

A conventional method to analyze two mobility-lifetime products is based on Hecht equation and α particle spectrum measured at various bias voltages [2]. However, the α particle method cannot evaluate the $(\mu\tau)_h$ product in many cases because a semiconductor detector operating at positive bias voltages frequently produces the energy spectrum without peaks.

In this study, a new approach with gamma-rays instead of α particle was carried out to solve the determination difficulty of the $(\mu\tau)_h$ product. The special relation between the two mobility-lifetime products, called a "Nural equation", was developed to include the contribution of holes and to accurately obtain each parameter based on the Hecht equation. In order to confirm the possibility of this approach, the mobility-lifetime products of the electron-hole pair in a 5×5×2 mm^3 CZT detector were determined by fitting a gamma-ray peak position as a function of biased voltage to the modified equation.

2. Maximum Charge Collection Efficiency

Assuming that the electric field is constant within semiconductor material, when gamma-ray is incident to the negative electrode, the charge collection efficiency is given by Hecht equation (see Eq. (1)). Where x is the distance from the negative electrode to the position at which the charge carriers are produced, and D is the detector thickness. The λ_e and λ_h are the mean free paths of electrons and holes, respectively ($\lambda_e = E\times(\mu\tau)_e$ and $\lambda_h = E\times(\mu\tau)_h$). E is the strength of the electric field in semiconductor detector, and $\tau_{e,\,h}$ and $\mu_{e,\,h}$ are their lifetimes and mobilities, respectively.

In the Hecht equation, there is a specific position that includes important physical meaning such as producing maximum charge collection efficiency. Hence, when the gamma-rays are incident to the negative electrode, the specific position ($x_{\max}$) is calculated by differentiating Eq. (1) with respect to x (see Eq. (2)). And then, the maximum charge collection efficiency is finally derived by substituting Eq. (2) to x in Eq. (1), as shown in Eq. (3).

$$\eta(x) = \left\{ \begin{array}{l} \dfrac{\lambda_e}{D}\left(1-\exp\left(-\dfrac{D-x}{\lambda_e}\right)\right)+\dfrac{\lambda_h}{D}\left(1-\exp\left(-\dfrac{x}{\lambda_h}\right)\right) \\ \dfrac{1}{D}\left(\lambda_e+\lambda_h-\lambda_e\exp\left(-\dfrac{D-x}{\lambda_e}\right)-\lambda_h\exp\left(-\dfrac{x}{\lambda_h}\right)\right) \end{array} \right. \qquad (1)$$

$$x_{\max} = \frac{D\lambda_h}{\lambda_e+\lambda_h} = \frac{D(\mu\tau)_h}{(\mu\tau)_e+(\mu\tau)_h} \qquad (2)$$

$$\eta(x_{\max}) = \left\{ \begin{array}{l} \dfrac{1}{D}(\lambda_e+\lambda_h)\left(1-\exp\left(-\dfrac{D}{\lambda_e+\lambda_h}\right)\right) \\ \dfrac{1}{D}E(\mu\tau)_{sum}\left(1-\exp\left(-\dfrac{D}{E(\mu\tau)_{sum}}\right)\right) ;\ E(\mu\tau)_{sum}=\lambda_e+\lambda_h \end{array} \right. \qquad (3)$$

where $(\mu\tau)_{sum}$ is the sum of the mobility-lifetime products for the electron and hole. It is found that the equation representing the maximum charge collection efficiency (Eq. (3)) is not affected by the incident direction of gamma-ray and is relatively very simple in form.

3. Modified Single Parameter Hecht Equation

Although the gamma-ray has a large number of possible interaction mechanisms in matter, only three reactions play an important role in radiation measurements: photoelectric effect, Compton scattering, and pair production. The sum of these probabilities is the probability per unit path length that the gamma-ray is removed in matter (μ = τ (photoelectric) + σ (Compton) + κ (pair)). The incident gamma-ray can also be characterized by its mean free path (λ_γ) defined as the average distance traveled in the detection material before an interaction takes place, as follows:

$$\lambda_\gamma = \frac{\int_0^\infty x e^{-\mu x}dx}{\int_0^\infty e^{-\mu x}dx} = \frac{1}{\mu} \qquad (4)$$

In the case of semiconductor detectors, it can be assumed that incident gamma-rays are absorbed by only one reaction because the probability of a photoelectric effect is much higher than other reactions at low energy ranges ($E_\gamma \leq 250$ keV). Based on this assumption, most charge carriers are produced at mean free path of incident gamma-ray, and the charge collection efficiency at this position determines the energy range of full-energy peak in gamma-ray energy spectrum.

At this time, the correction factor (x) is defined as the rate of $x_{\max}$ distance to the mean free path of the incident gamma-ray. Through the introduction of the correction factor, additional equation with respect to $x_{\max}$ is obtained by the following Eq. (5). However, Eq. (5) does not mean that the actual mean free path of radiation source used in the experiment is controlled by the special device or specific treatment, because the mean free path of incident gamma-ray depends only on the radiation energy and interacting materials.

$$x_{\max} = \lambda_\gamma \times \varepsilon = \frac{\varepsilon}{\mu} \tag{5}$$

Also, the maximum charge collection efficiency different with Eq. (2) is derived by substituting Eq. (5) to x in Eq. (1), as shown in Eq. (6), and therefore, the Nural equation (see Eq. (7)) was newly derived by forming simultaneous equation based on the Equation (3) and (6). Although, mean free path of radiation source used in the experiment is different with efficiency, the mobility-lifetime products of the electron and hole can be derived by Nural Equation and fitting a modified single parameter Hecht equation (see Eq. (8)) to the peak variations of measured energy spectrums as a function of applied bias voltages.

$$\eta(x_{\max}) = \frac{1}{D}\left(\lambda_e + \lambda_h - \lambda_e \exp\left(-\frac{D\mu - \varepsilon}{\mu\lambda_e}\right) - \lambda_h \exp\left(-\frac{\varepsilon}{\mu\lambda_h}\right)\right) \tag{6}$$

$$(D\mu - \varepsilon)(\lambda_e + \lambda_h) = D\mu\lambda_e$$

$$; \begin{cases} \lambda_h = \dfrac{\varepsilon\lambda_e}{(D\mu - \varepsilon)} \\ (\mu\tau)_h = \dfrac{\varepsilon(\mu\tau)_e}{(D\mu - \varepsilon)} \end{cases} \tag{7}$$

$$\eta(V) = \frac{1}{D}\left(\lambda_e + \frac{\varepsilon\lambda_e}{D\mu - \varepsilon}\right)\left(1 - \exp\left(-\frac{D}{\lambda_e + \dfrac{\varepsilon\lambda_e}{D\mu - \varepsilon}}\right)\right)$$

$$= \frac{(\mu\tau)_e \mu}{D\mu - \varepsilon} E\left(1 - \exp\left(-\frac{D\mu - \varepsilon}{E(\mu\tau)_e \mu}\right)\right) \tag{8}$$

4. Performance Evaluation of New Approach

A commercial grade planar CZT detector (5×5×2 mm^3) manufactured by eV Products was selected to analyze the charge transport properties. In order to evaluate accuracy of the new approach, the selected detector was irradiated with low energy gamma-rays (E_γ = 59.5 keV) emitted from ^{241}Am isotope through the front surface of the detector. The series of the experiments performed in this study progressed as the bias voltage was changed from -200 V to 200 V. As shown in **Figure 1**, there is some deviation (standard deviation = ± 4.64×10^{-5} cm^2/V) in two mobility-lifetime products derived from the new approach, and average values of $(\mu\tau)_e$ and $(\mu\tau)_h$ are evaluated with 1.44×10^{-3} cm^2/V and 7.56×10^{-5} cm^2/V, respectively.

As shown in **Figure 2**, the measured result of 59.5 keV gamma-rays is compared with the energy spectrum simulated by new approach, in order to confirm the accuracy of derived mobility-lifetime products by this work. It can be seen that the energy spectrum derived from the new approach agrees well with experimental results in an energy range from 0 – 70 keV. Therefore, it is recognized that the new method using low energy gamma-rays is useful to investigate the transport properties of a semiconductor detector.

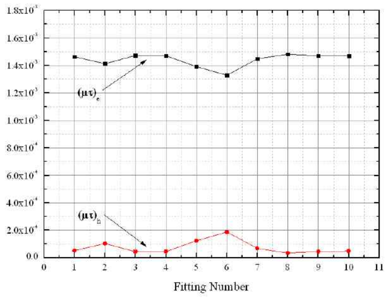

Fig. 1. Change of Two Mobility-lifetime Products derived from the New Approach

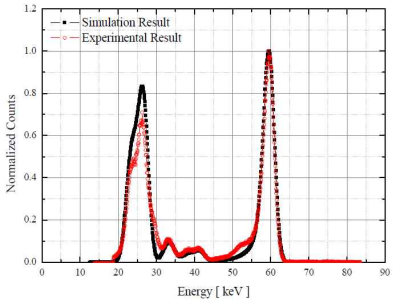

Fig. 2. A Comparison of Experimental and Simulated Results by Considering Transport Properties of the Electron-hole Pair

5. Conclusions

By developing the relation between the two mobility-lifetime products based on Hecht equation, a new approach based on a response to gamma-ray was introduced to obtain each parameter. The electron and hole mobility-lifetime products in a 5×5×2 mm^3 CZT detector were simply evaluated from the modified single parameter Hecht equation derived in this work. From this result, it is expected that the new approach is particularly useful to accurately evaluate the (μτ) product of the electron and hole in semiconductor detector.

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