Development of Sensitivity Analysis Code for Fast Reactor Application by Using Perturbation Theory

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1. Introduction

Sensitivity and uncertainty have played important role in a validation of fast reactor analysis code, in particular against the critical and reactor physics experiments. This is mainly because neutron cross sections of transuranic isotopes at the high energy are relatively uncertain and the impacts on the integral parameters and reactivity effects are considerable in fast reactor design.

The sensitivity analysis should be also accompanied with uncertainty analysis, cross section adjustment and justification of similarity between experiment and design during the validation procedure of fast reactor analysis code.

The perturbation theory has been widely used as a methodology to calculate the sensitivity coefficients in order to avoid a number of direct calculations.[1-2] This paper describes the general features of the developed sensitivity and perturbation analysis code system and the results of its verification.

2. Methods and Results

2.1 Perturbation integral and sensitivity coefficient

Boltzman neutron transport equation can be written as follows:

$$\hat{\Omega}\cdot\nabla\psi+\Sigma_t\psi=\int_{E'}dE'\int_{\hat{\Omega}'}d^2\hat{\Omega}'\Sigma_s\left(\vec{r},E'\to E,\hat{\Omega}'\to\hat{\Omega}\right)\psi\left(\vec{r},E',\hat{\Omega}'\right)+\frac{1}{k}\frac{\chi}{4\pi}\int_{E'}dE'\int_{\hat{\Omega}'}d^2\hat{\Omega}'\nu\Sigma_f\left(\vec{r},E',\hat{\Omega}'\right)\psi\left(\vec{r},E',\hat{\Omega}'\right) \quad (1)$$

The neutron transport equation can be expressed as following matrix and vector form for the forward and adjoint solutions.

$$\left(\mathbf{A}-\frac{\mathbf{F}}{k}\right)\boldsymbol{\psi}=0\,,\;\left(\mathbf{A}^+-\frac{\mathbf{F}^+}{k}\right)\boldsymbol{\psi}^+=0 \quad (2)$$

Based on the Eq. (2) the sensitivity coefficient for reactivity can be defined as follows based on the perturbation theory.

$$S=\frac{dk}{k}\frac{\sigma_{r,x,g,n}}{\delta\sigma_{r,x,g,n}}=-k\frac{\left\langle\boldsymbol{\psi}^+,\left(\mathbf{A}-\frac{\mathbf{F}}{k}\right)_{\sigma_{r,x,g,n}}\boldsymbol{\psi}\right\rangle}{\left\langle\boldsymbol{\psi}^+,\mathbf{F}\boldsymbol{\psi}\right\rangle} \quad (3)$$

In order to calculate the sensitivity coefficient by isotope and reaction type for corresponding neutron energy, following five perturbation integrals will be used. Eq. (4) indicates the denominator term of Eq. (3), which is integrated over the whole reactor system and neutron energy groups.

$$EF=\int_{core}\left(\sum_{g'=1}^{G}\chi_{g'}\phi_{g'}^+\right)\cdot\left(\sum_{g=1}^{G}\nu\Sigma_{fg}\phi_g\right)dV \quad (4)$$

Eq. (5) is the integral involved in the left hand side of Eq. (1), Eq. (6) associated with the first term of right hand side of Eq. (1). Eq. (7) and Eq. (8) are integrals corresponding to the integration of second term of right hand side of Eq. (1). The perturbation integrals shown in Eq. (5) to Eq. (8) should be multiplied by –k/*EF*

$$IR_g=\int\left(\sum_{i=1}^{N_\Omega}w(\hat{\Omega}_i)\psi(\hat{\Omega}_i)\psi^+(-\hat{\Omega}_i)\right)dV \quad (5)$$

$$IS_{g,g'}^l=\sum_{m=\frac{l(l+1)}{2}}^{m=\frac{l(l+3)}{2}}\int\psi_{m,g}\psi_{m,g'}^+dV \quad (6)$$

$$IF_g=\int_V\phi_g^+\left(\sum_{g'=1}^{G}\nu\Sigma_{fg'}\phi_{g'}\right)dV \quad (7)$$

$$IP_g=\int_V\phi_g\left(\sum_{g'=1}^{G}\chi_{g'}\phi_{g'}^+\right)dV \quad (8)$$

Utilizing these perturbation integral of neutron transport equation, the sensitivity coefficient for each isotope and reaction type can be expressed into the following equations.

$$\text{Fission Spectrum } S(\chi_g)=\chi_g IF_g \quad (9)$$

$$\text{Capture } S(\sigma_{\gamma,g})=-\Sigma_{\gamma,g}IR_g \quad (10)$$

$$\text{Neutron production } S(\nu_g)=\nu\Sigma_{f,g}IP_g \quad (11)$$

$$\text{Scattering } S(\sigma_{el,g})=\sum_{l=0}^{L}\sum_{g'=1}^{G}\Sigma_{x,g\to g'}^l IS_{gg'}-\Sigma_{x,g}IR_g, \quad (12)$$

$$x=\text{elastic, inelastic, n2n}$$

2.2 Development of Sensitivity Analysis Code

Sensitivity analysis code, called as APSTRACT (Analyzer of Perturbation and Sensitivity with TRAnsport CalculaTion), has been developed at KAERI in the framework of the validation of fast reactor analysis code system. The forward and adjoint solution is taken from the results of TWODANT 2-D S_N neutron transport equation solver.

Fig. 1 shows the interface between TWODANT and APSTRACT codes. The forward and adjoint angular flux file contained into RAFLXM and AAFLXM is directly transferred to APSTRACT and geometry and region-wise atomic number density are given as input. In addition to the interfaces with TWODANT code results, the APSTRACT separately reads scattering cross section from multi-group master library in

MATXS format. This is because the current ISOTXS produced from the TRANSX code only contains total scattering cross section matrices. Accordingly, elastic, inelastic and n2n scattering cross sections are added into the original ISOTXS file by directly reading from MATXS file.

The APSTRACT code produces two interface files, perturbation integral (PTBINT) and sensitivity coefficient (SENSCF) files. The perturbation integral file can be used for another separate calculation of perturbation or sensitivity analysis when the forward and adjoint solutions are not changed. The sensitivity coefficient file will be used for the uncertainty analysis with covariance matrix or the adjustment of cross section which is projected during next two years for development.

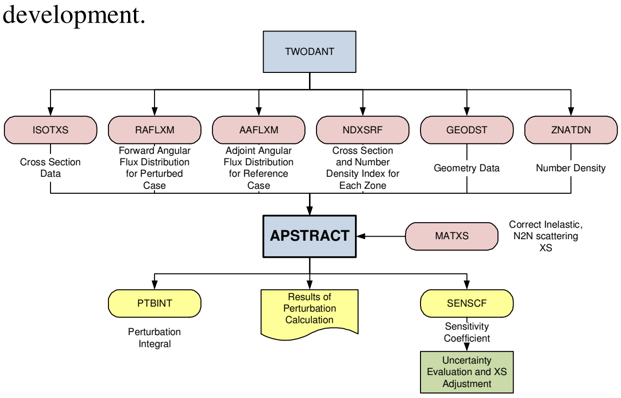

Fig. 1. System diagram of sensitivity analysis code APSTRACT

2.3 Verification of sensitivity analysis code

The sensitivity analysis code was verified against the results of direct perturbation calculation and sensitivity coefficient calculated by the ERANOS code system.

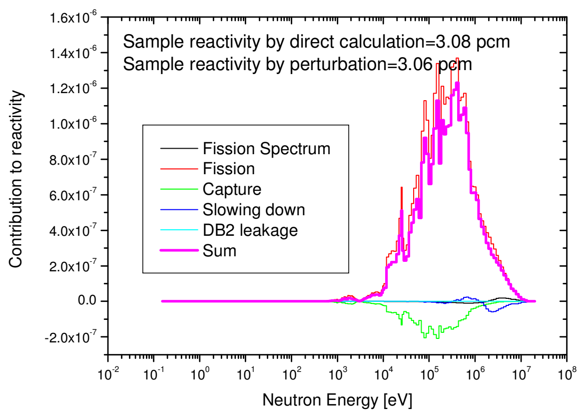

Fig. 2. Comparison of reactivity change arising from a U-235 sample insertion to the central core region of the BFS-75-1 experiment with a direct perturbation calculation

Fig. 2 shows comparison of APSTRACT results with those from a direct calculation. This is the case that a U-235 sample is inserted into the central core region of the BFS-75-1 critical experiment. As shown in the figure, the reactivity change calculated by APSTRACT is well agreed with that of the direct calculation. The contribution of fission reaction is compensated by that of radiative capture reaction and group-wise contribution to reactivity change can be seen in the figure.

The integral sensitivities are compared with those from sensitivity and perturbation analysis module implemented into ERANOS2.1 code system as shown in Table I. The most sensitive part to k-effective was the number of neutron production by fission (ν). Its effect to reactivity is 0.83 %Δk/k arising from 1 % change of ν. The maximum difference of integrated sensitivity is less than 1 % as listed in the table.

Table I: Comparison of BFS-75-1 whole reactor U-235 integrated sensitivity

	Nu (ν)	Fission	Capture
ERANOS	8.3138E-01	5.3153E-01	-6.5164E-02
APSTRACT	8.2906E-01	5.2687E-01	-6.5373E-02
% difference	-0.28	-0.86	0.32

3. Conclusions

A sensitivity analysis code, APSTRACT was developed for fast reactor uncertainty analysis and data adjustment. This code utilize the neutron flux solution from TWODANT S_N transport code and is verified against the results of a direct perturbation calculation and ERANOS2.1 code system[3]. As a result of verification, the perturbation module of the APSTRACT code well reproduces the result of direct calculation and the integrated sensitivity is agreed with ERANOS code within 1 %.

The sensitivity coefficient produced from the APSTRACT code will be used for cross section uncertainty evaluation and projected data adjustment module.

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