Transactions of the Korean Nuclear Society Spring Meeting
Pyeongchang, Korea, May 27-28, 2010

A Contact Force Prediction Method for Nuclear Fuel Rod Supported by Multiple Spacers

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1. Introduction

In the field of nuclear industry, the structure with a dynamic contact can be often seen. For example, U tubes in the heat exchanger and fuel rods in the reactor are representative, and predicting the dynamic behavior of the tubes or fuel rods is critical to evaluate whether the structural integrity against wear is robust enough.

For the fuel rod, due to thermal relaxation of the elastic supports and creep-down of the fuel rod cladding, the gap develops between the fuel rod and the supports. Therefore the dynamic impact caused by turbulent coolant flow is unavoidable. A degree of freedom which is free in the fuel rod becomes restrained at a certain time when the gap closes. Usually the system is modeled as a beam with extra external forces or pseudo forces which simulate a dynamic contact condition[1~2]. That is when the fuel rod contacts the support, the reaction force is developed and the force is regarded as an extra external force. A time domain solution can be obtained by direct implicit integration of the governic equation using the Newmark method, but the predicted solution may not be reliable because of the unknown contact force.

This paper proposes continuous and differentiable contact force model that is required to calculate its tangent stiffness.

2. Methods and Results

2.1 Equation of Motion

Regarding the contacting force acting on the beam as an external force, the equation of motion can be written as

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]_L\{u\}=\{f^{ext}\}+\{f^C\} \quad (1)$$

,where $\{u\}$ and $[M]$ denote displacement vector and mass matrix, respectively. $[K]_L$, $[C]$, and $\{f^{ext}\}$ are linear stiffness matrix, damping matrix, and external force vector, respectively. For linear supports, the last term in Eq.(1), $\{f^C\}$, is the contact force of which a component is defined as :

$$f_i^C=\begin{cases}-k_i^C\left(u_i-g_i\right) & , if\ u_i>g_i \\ 0 & , if\ u_i\le g_i\end{cases} \quad (2)$$

,where g_i and k_i^C denote i-th gap distance and i-th support spring constant, respectively. Note that Eq.(2) is effective only for the upper supports, the supports above the beam. For the lower supports, the contact force is active only when the displacement is lower than the gap.

Using the Newmark method, Eq.(1) can be expressed as

$$[\mathbf{K}]\{u\}_{t+\Delta t}=\left(\{f^{ext}\}_{t+\Delta t}+\{f^C\}_{t+\Delta t}+\{F\}\right) \quad (3)$$

,where $[\boldsymbol{K}]$ is dynamic stiffness which is defined as :

$$[\mathbf{K}]=[K]_L+\frac{1}{\beta\Delta t^2}[M]+\frac{\gamma}{\beta\Delta t}[C] \quad (4)$$

$\{F\}$ in Eq.(3) is another force vector due to the inertia and damping at time t. Usually, considering stability of the integration, 1/4 and 1/2 are preferred for β and γ, respectively.

Based on Eq.(4), the solution can be found, but it should be noted that the contact force, $\{f^C\}$, is unknown at time $t+\Delta t$. Johansson[3] tried to solve the problem using the Taylor expansion of the contact force. That is :

$$\{f^C(u_{t+\Delta t})\}=\{f^C(u_t)\}+\frac{\partial\{f^C\}}{\partial\{u\}}\{u_{t+\Delta t}-u_t\} \quad (5)$$

This expression is meaningful when the contact force is differentiable, but the contact force model of Eq.(2) is not differentiable at $u_i=g_i$. The solid line in Fig. 1 shows the graphical representation of the force model, and it is continuous but the slope is not obviously continuous.

2.2 Approximated Contact Force Model

To relieve the drawback, an approximated contact force can be defined as

$$f_i^C=\begin{cases}-k_i^C\left(u_i-g_i\right) & , \quad if\ u_i>g_i+\varepsilon \\ -\dfrac{k_i^C}{4\varepsilon}\left(u_i-\left(g_i-\varepsilon\right)\right)^2 & , \quad if\ g_i-\varepsilon\le u_i\le g_i+\varepsilon \\ 0 & , \quad if\ u_i<g_i-\varepsilon\end{cases} \quad (6)$$

,where ε denotes a reasonably small positive number. Actually, Eq.(6) means a continuously varying contact force shown in Fig.1. Clearly, the contact force is not only continuous, but also the slope is continuous over the entire displacement domain. With Eq.(6), substituting Eq.(5) to Eq.(3), it leads to

$$([\mathbf{K}]-[T])\{u\}_{t+\Delta t}=\left(\{f^{ext}\}_{t+\Delta t}+\{f^C\}_t-[T]\{u\}_t+\{F\}\right) \quad (7)$$

,where $[T]$ is tangential stiffness of the contact force, $\frac{\partial\{f^C\}}{\partial\{u\}}$. Considering that $[\mathrm{K}]$ is linear and constant while $[T]$ is nonlinear, Eq.(7) can be rewritten as :

$$\{u\}_{t+\Delta t} = [\mathrm{K}]^{-1}\left(\{P\}_{t+\Delta t} + [T]\{u\}_{t+\Delta t}\right) \tag{8}$$

,where $\{P\}$ means the terms in the right hand side of Eq.(9). An approximated solution, especially for a nonlinear structure, can be found using an iterative computation method.

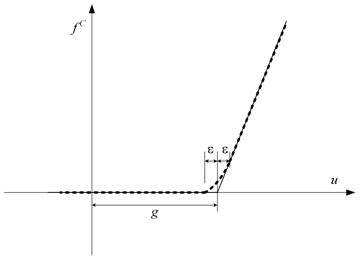


Fig. 1 Approximated contact force (dotted line)

2.3 Simulation Example

It is assumed that a harmonic external force with 10 Hz frequency and 2 N of the magnitude is applied at the center of every span. Rayleigh damping which is proportional to the stiffness matrix is used, and the coefficient of the proportional damping is set to 2.7×10^{-4} to damp out all the high natural frequencies that are bigger than the 50-th natural frequency. The time increment is set to 2×10^{-5} second. Fig. 2 shows the deflected shape for the first few seconds, and the maximum amplitude at time 0.012 second exists in the first span which is the longest. The load-deflection curves for the 5-th mid grid are shown in Fig.3. The magnified view of the load-deflection curve, Fig. 3, shows that multiple lines exist. It is due to the fact that the predicted motion does not exactly coincide with the given gap distance. The deviation is negligible, but when the modified contact force model is used, though the ideal load-deflection path cannot be kept, such an error can be minimized.

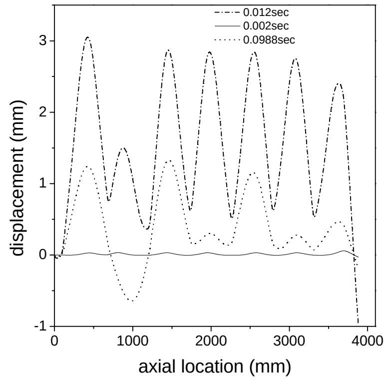
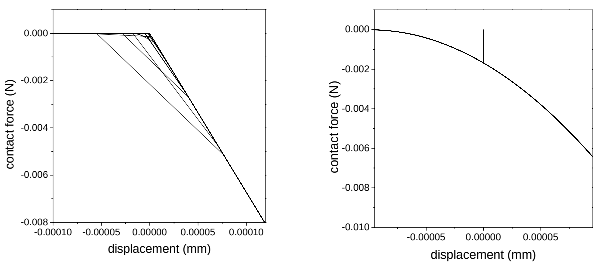


Fig. 2 Deflected shapes



(a) existing method (b) suggested method

Fig. 2 Contact force estimation results

3. Conclusions

For the structure accompanied by contact phenomenon, dynamic contact force model needs to be defined. Solutions can be obtained using the Newmark method, which requires the contact force a priori. An unknown contact force in current time can be estimated using Taylor expansion, and therefore a dynamic contact model should be differentiable over the displacement field. Such a model was suggested in the work, and an approximated solution can be found.

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