Improvement of a Stochastic Model Applied to Mono-energetic Space-time Nuclear Reactor Kinetics

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1. Introduction

In a previous study, a simplified stochastic model (called the stochastic space-dependent kinetics model or SSKM) based on the forward stochastic model in stochastic kinetics theory and the Itô stochastic differential equations was proposed for the analysis of mono-energetic space-time nuclear reactor kinetics in one dimension [1-5]. However, the SSKM was derived for just one delayed-neutron precursor group. For that reason, the SSKM has been enhanced and evaluated in this study for *M* precursor groups.

2. Methods and Results

The SSKM will be extended for *M* delayed-neutron precursor groups as given in the following section.

2.1 Extension of the SSKM to M precursor groups

As the reactor is partitioned into *I* space cells, the SSKM for one spatial dimension, one energy group and *M* precursor groups is developed as follows:

$$\frac{d}{dt}\begin{bmatrix} \bar{n}_i(t) \\ \bar{c}_{i1}(t) \\ \bar{c}_{i2}(t) \\ \dots \\ \bar{c}_{iM}(t) \end{bmatrix} = \hat{\mathrm{A}} \begin{bmatrix} \bar{n}_i(t) \\ \bar{c}_{i1}(t) \\ \bar{c}_{i2}(t) \\ \dots \\ \bar{c}_{iM}(t) \end{bmatrix} + \begin{bmatrix} q_i(t) \\ 0 \\ 0 \\ \dots \\ 0 \end{bmatrix} + \hat{\mathrm{B}}^{\frac{1}{2}} \frac{d\vec{W}(t)}{dt} \qquad (1)$$

where

$\bar{n}_i(t)$ is the number of neutrons in space cell *i* (*i* = 1, 2, …, *I*),
$\bar{c}_{im}(t)$ is the number of *m*-type delayed-neutron precursors in space cell *i* (*m* = 1, 2, …, *M*),
$S_i(t)$ is the neutron source rate in space cell *i*,
$\Lambda_{ci}(t)$ is the capture frequency per neutron in space cell *i*,
$\Lambda_{fi}(t)$ is the fission frequency per neutron in space cell *i* ($\Lambda_{fi} = \upsilon\Sigma_f$, where υ is the neutron speed and Σ_f is the macroscopic fission cross section),
λ_m is the decay constant for precursor type *m*,
β'_m is the average ratio of the number of *m*-type precursors to the number of prompt neutrons produced in a fission ($\beta' = \sum_m \beta'_m$),
$l_{ii'}(t)$ represents the frequency per neutron at which neutrons in space cell *i* will diffuse into space cell *i'*,
$\bar{\nu}_p = (1-\beta)\bar{\nu}$, where $\bar{\nu}$ is the average number of neutrons (prompt and delayed) per fission,
$\beta' = \beta/(1-\beta)$, where β is the total delayed neutron fraction,

$$\hat{\mathrm{A}} = \begin{bmatrix} -d + b(\bar{\nu}_p - 1) & \lambda_1 & \lambda_2 & \dots & \lambda_M \\ \beta'_1\bar{\nu}_p b & -\lambda_1 & 0 & \dots & 0 \\ \beta'_2\bar{\nu}_p b & 0 & -\lambda_2 & \dots & 0 \\ \dots & & & & \\ \beta'_M\bar{\nu}_p b & 0 & 0 & \dots & -\lambda_M \end{bmatrix},$$

$$\hat{\mathrm{B}} = \begin{bmatrix} \xi & a_1 & a_2 & \dots & a_M \\ a_1 & r_1 & b_{2,3} & \dots & b_{2,M} \\ a_2 & b_{3,2} & r_2 & \dots & b_{3,M} \\ \dots & & & & \\ a_{M-1} & b_{M-1,2} & \dots & r_{M-1} & b_{M-1,M} \\ a_M & b_{M,2} & \dots & b_{M,M-1} & r_M \end{bmatrix},$$

$$b \equiv \Lambda_{fi}(t),$$

$$d \equiv \Lambda_{ci}(t),$$

$$\xi = (d + b(\bar{\nu}_p - 1)^2)\bar{n}_i(t) + \sum_{m=1}^{M} \lambda_m \bar{c}_{im}(t) + Q_i(t),$$

$$a_m = \beta'_m\bar{\nu}_p b(\bar{\nu}_p - 1)\bar{n}_i(t) - \lambda_m\bar{c}_{im}(t),$$

$$b_{m,m'} = (\beta'_m\bar{\nu}_p)(\beta'_{m'}\bar{\nu}_p) b\bar{n}_i(t),$$

$$r_m = (\beta'_m\bar{\nu}_p)^2 b\bar{n}_i(t) + \lambda_m\bar{c}_{im}(t),$$

$$q_i(t) = \sum_{i'=i-1}^{i+1} [l_{i'i}(t)\bar{n}_{i'}(t) - l_{ii'}(t)\bar{n}_i(t)] + S_i(t),$$

$$Q_i(t) = l_{i+1i}(t)\bar{n}_{i+1}(t) + l_{i-1i}(t)\bar{n}_{i-1}(t) + [l_{ii+1}(t) + l_{ii-1}(t)]\bar{n}_i(t) + S_i(t),$$

and

$\vec{W}(t) = [W_1(t), W_2(t), \dots, W_{M+1}(t)]^T$ are Wiener processes.

The numerical solution for Eq. (1) is adopted from the previous study [5].

2.2 Numerical Validation

A numerical experiment is performed by using MATLAB and FORTRAN in order to validate the extended model for *M* precursor groups through comparison with the Monte Carlo calculations.

The experiment simulates a positive step reactivity insertion within a homogeneous slab reactor which is partitioned into five space cells (I = 5). The values λ_m = [0.0127, 0.0317, 0.115, 0.311, 1.4, 3.87], β_m = [0.000266, 0.001491, 0.001316, 0.002849, 0.000896, 0.000182], β = 0.007, $\bar{\nu}$ = 2.5, the fission rate $\upsilon\Sigma_f$ = 20000/sec, the absorption rate $\upsilon\Sigma_a$ = 49850/sec, the slab thickness H = 150 cm, the diffusion coefficient D_i = 0.05 cm, and $\upsilon D_i B_{gi}^2 = 100$/sec ($B_{gi}^2 = (\pi I/H)^2$ is the geometric buckling for space cell *i*, where *i* = 1, 2, …, *I*) are considered (note that $l_{ii'}(t) = l_{i'i}(t) = \frac{\upsilon D_i B_{gi}^2}{2}$ for the uniform slab reactor of interest, where $i' = i$-1, i+1). The uniform external source rate S_i = 6666.7/sec and the homogeneous initial conditions $[\bar{n}_i(0),\ \bar{c}_{i1}(0),\ \bar{c}_{i2}(0),\ \ldots,\ \bar{c}_{i6}(0)]$ = [33.3, 34908.1, 78391.2, 19072.5, 15268.0, 1066.7, 78.4] are adopted for each inner space cell (i = 2, 3, 4) while the vacuum boundary conditions are applied to the two outermost space cells (i = 1, 5). For the SSKM calculations, 40 time intervals for a time interval of length 0.1 seconds are used. The number of trials for both SSKM and Monte Carlo calculations is 5000, and it is found that the SSKM is more than one hundred times faster than the Monte Carlo method in this sample calculation.

Good agreement between two different calculation procedures at time t = 0.1 seconds is shown in Table I where the mean values of neutron and precursor populations ($E(\bar{n}_i)$ and $E(\sum_{m=1}^{6} \bar{c}_{im})$; i = 1, 2, …, 5 and m = 1, 2, …, 6) are listed with their standard deviations ($\sigma(\bar{n}_i)$ and $\sigma(\sum_{m=1}^{6} \bar{c}_{im})$; i = 1, 2, …, 5 and m = 1, 2, …, 6). Additionally, Figs. 1-2 show the mean neutron and precursor populations for 5000 trials modeled by the SSKM. As a result, it can be seen that the stochastic neutron distribution is generally more disperse than the stochastic precursor distribution at the same spatial location within the reactor of interest.

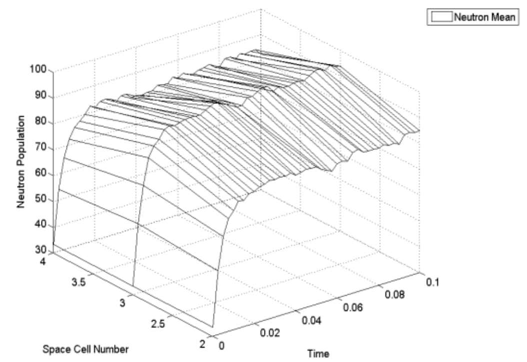

Fig. 1. Mean neutron level for a uniform slab reactor which is partitioned into a set of space cells with time in seconds.

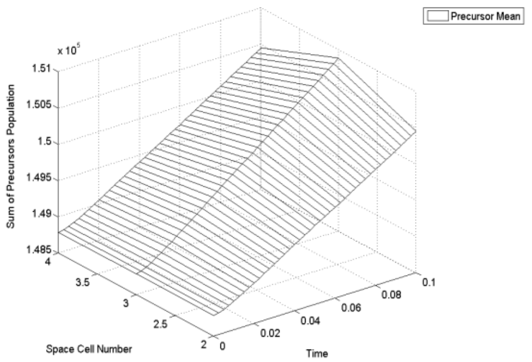

Fig. 2. Mean precursor level for a uniform slab reactor which is partitioned into a set of space cells with time in seconds.

Table I: A comparison of mean neutron level and mean fission product level with their standard deviations calculated at time t = 0.1 seconds using the SSKM and Monte Carlo method for a positive step reactivity insertion within a uniform slab reactor.

Space cell	Estimated values	Monte Carlo	SSKM	Relative difference
2	$E(\bar{n}_2(0.1))$	83.26	84.81	1.86%
	$\sigma(\bar{n}_2(0.1))$	101.30	78.70	-
	$E(\sum_{m=1}^{6} \bar{c}_{2m}\ (0.1))$	150400	150450	0.03%
	$\sigma(\sum_{m=1}^{6} \bar{c}_{2m}\ (0.1))$	728.4	947.14	-
3	$E(\bar{n}_3(0.1))$	93.82	93.84	0.02%
	$\sigma(\bar{n}_3(0.1))$	108.0	83.31	-
	$E(\sum_{m=1}^{6} \bar{c}_{3m}\ (0.1))$	150700	150880	0.12%
	$\sigma(\sum_{m=1}^{6} \bar{c}_{3m}\ (0.1))$	761.4	1042.7	-
4	$E(\bar{n}_4(0.1))$	81.96	82.52	0.68%
	$\sigma(\bar{n}_4(0.1))$	101.30	75.76	-
	$E(\sum_{m=1}^{6} \bar{c}_{4m}\ (0.1))$	150300	150450	0.08%
	$\sigma(\sum_{m=1}^{6} \bar{c}_{4m}\ (0.1))$	720.8	943.2	-
Total	$Sum(E(\bar{n}_i(0.1)))$	259.04	261.17	0.82%
	$Sum(E(\sum_{m=1}^{6} \bar{c}_{im}\ (0.1)))$	451400	451780	0.08%

Note: Relative difference (%) = 100*(SSKM - Monte Carlo)/Monte Carlo

3. Conclusions

In this work, the stochastic space-dependent kinetics model (SSKM) was developed to M delayed-neutron precursor groups. The numerical results showed that the SSKM agrees well with the Monte Carlo method. Again, the SSKM was found to provide a fast calculation method in comparison with the Monte Carlo runs. It is also noteworthy to recall that the SSKM can be considered as a generalization of the stochastic point-kinetics equations by Hayes and Allen.

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