Extension of Analytic Function Expansion Nodal (AFEN) Method for Multigroup Simplified P_3 (SP_3) Equations

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1. Introduction

Most existing methods to analyze reactor cores are based on the solution of the neutron diffusion equation with nodal methods. However, improvement of the diffusion model may be required to increase accuracy of power distribution and k_{eff} due to the new types of fuels such as MOX and new types of reactors. Recently, SP_3 approximation (to transport equation) [1] is in the limelight since it is expected to be more accurate than the diffusion equation, and it has a very similar form with diffusion equation which may lead to easy implementation in conventional nodal diffusion codes.

In this work, the AFEN method has been reformulated for SP_3 equations and implemented in a version of the AFEN method code COREDAX [2] for cubic geometry. The COREDAX code with SP_3 was tested on transport benchmark problems, and the reference values are obtained by the DANTSYS code which is based on discrete ordinates transport method. The results show significant improvement in power distribution and k_{eff} compared to the diffusion method.

2. Basic Theory and Method

2.1 The AFEN Method

The AFEN formulation in the (x,y,z) coordinates system starts from the following multigroup diffusion equations in a homogenized node :

$$-\nabla^2\vec{\phi}(x,y,z)+[\Lambda]\vec{\phi}(x,y,z)=0, \qquad (1)$$

where

$$[\Lambda]=[D]^{-1}\left([\Sigma]-\frac{1}{k_{eff}}[\chi][\nu\Sigma_f]\right),$$

and all the notations are standard.

Following the AFEN methodology [3], the solution of Eq. (1) for node n is expressed in the following form :

$$\begin{aligned}\overrightarrow{\phi^n}(x,y,z)=&\overrightarrow{E^n}+\overrightarrow{\varphi^n}(x,y,z)+\overrightarrow{\varphi^n}(y,x,z)\\&+\overrightarrow{\varphi^n}(z,x,y),\end{aligned} \qquad (2)$$

where

$$\begin{aligned}\overrightarrow{\varphi^n}(x,y,z)=&\sinh(\sqrt{\Lambda^n}x)(\overrightarrow{A^n_{0x}}+y\overrightarrow{A^n_{1x}}+z\overrightarrow{A^n_{2x}})\\&+\cosh(\sqrt{\Lambda^n}x)(\overrightarrow{B^n_{0x}}+y\overrightarrow{B^n_{1x}}+z\overrightarrow{B^n_{2x}}).\end{aligned}$$

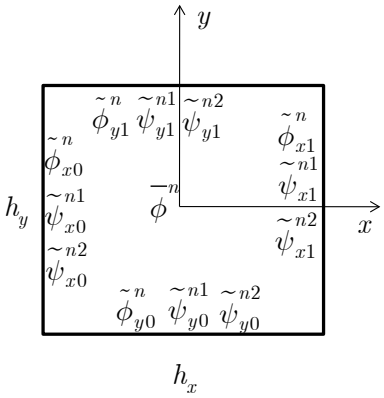

Fig. 1. Geometry of node n in x-y plane.

The 19 coefficients in the flux expansion Eq. (2) are to be determined in terms of 19 nodal unknowns, a node average flux, six interface fluxes, and twelve interface flux moments as follows :

$$\bar{\phi}^n=\frac{1}{h_x h_y h_z}\int_{-h_z/2}^{h_z/2}\int_{-h_x/2}^{h_x/2}\int_{-h_y/2}^{h_y/2}\vec{\phi}(x,y,z)dxdydz,$$

$$\tilde{\phi}^n_{x0}=\frac{1}{h_z}\frac{1}{h_y}\int_{-h_z/2}^{h_z/2}\int_{-h_y/2}^{h_y/2}\vec{\phi}(-\frac{h_x}{2},y,z)dydz,$$

$$\tilde{\psi}^{n1}_{x0}=\frac{1}{h_z}\frac{1}{h_y}\int_{-h_z/2}^{h_z/2}\int_{-h_y/2}^{h_y/2}w_1(y,z)\vec{\phi}(-\frac{h_x}{2},y,z)dydz,$$

and

$$\tilde{\psi}^{n2}_{x0}=\frac{1}{h_z}\frac{1}{h_y}\int_{-h_z/2}^{h_z/2}\int_{-h_y/2}^{h_y/2}w_2(y,z)\vec{\phi}(-\frac{h_x}{2},y,z)dydz. \qquad (3)$$

In this paper, weighting functions, $w_1(y,z)$ and $w_2(y,z)$, are chosen as step functions :

$$w_1(y,z)=\begin{cases}-1 & \text{for } y<0\\ 1 & \text{for } y\geq 0\end{cases},$$

and

$$w_2(y,z)=\begin{cases}-1 & \text{for } z<0\\ 1 & \text{for } z\geq 0\end{cases}. \qquad (4)$$

To determine 19 nodal unknowns, we build 19 nodal equations for each node which are composed of a nodal balance equation and associated coupling equations. The coupling equations consist of six interface current continuity equations and twelve interface current moment continuity equations.

2.2 SP_3 Formulation

Following the SP_3 formulation [1], we obtain two diffusion-like equations as follows :

$$\begin{aligned} &-D_0^g\nabla^2\hat{\phi}_0^g+\Sigma_{r,0}^g\hat{\phi}_0^g-2\Sigma_{r,0}^g\hat{\phi}_2^g=S_0^g,\\ &-D_2^g\nabla^2\hat{\phi}_2^g-\frac{2}{5}\Sigma_{r,0}^g\hat{\phi}_0^g+\left[\Sigma_{r,2}^g+\frac{4}{5}\Sigma_{r,0}^g\right]\hat{\phi}_2^g \\ &=-\frac{2}{5}S_0^g, \end{aligned} \tag{5}$$

where

$$\begin{aligned} &\phi_l^g(\vec{r})=\int d\mu\psi(\vec{r},\mu)P_l(\mu),\\ &P_l(\mu)=l\,\text{th ordr Legendre polynomial},\\ &\hat{\phi}_0^g=\phi_0^g+2\phi_2^g,\ \hat{\phi}_2^g=\phi_2^g,\\ &\Sigma_{r,l}^g=\Sigma_t^g-\Sigma_{s,l}^{g\to g},\ \ l=0,1,2,3,\\ &D_0^g=\frac{1}{3\Sigma_{r,1}^g},\ \ D_2^g=\frac{9}{35\Sigma_{r,3}^g},\\ &S_0^g=\sum_{g'=1,g'\neq g}^{G}\Sigma_{s0}^{g'\to g}\phi_0^{g'}+\frac{\chi_g}{k_{eff}}\sum_{g'=1}^{G}\nu\Sigma_f^{g'}\phi_0^{g'}. \end{aligned}$$

Partial moments are given in terms of surface flux moments and current moments based on the Marshak-type boundary condition at a surface :

$$\begin{aligned} J_0^{g,\pm}&=\frac{1}{4}\hat{\phi}_0^g\pm\frac{1}{2}\vec{n}_s\cdot\vec{J}_0^g-\frac{3}{16}\hat{\phi}_2^g,\\ J_2^{g,\pm}&=-\frac{3}{80}\hat{\phi}_0^g\pm\frac{1}{2}\vec{n}_s\cdot\vec{J}_2^g+\frac{21}{80}\hat{\phi}_2^g, \end{aligned} \tag{6}$$

To be consistent with the AFEN formulation in diffusion format, we consider the following relations :

$$\begin{aligned} \overset{=g,\pm}{J}_0&\equiv\frac{1}{4}\hat{\phi}_0^g\pm\frac{1}{2}\vec{n}_i\cdot\vec{J}_0^g,\\ \overset{=g,\pm}{J}_2&\equiv\frac{1}{4}\hat{\phi}_2^g\pm\frac{1}{2}\vec{n}_i\cdot\vec{J}_2^g, \end{aligned} \tag{7}$$

which also satisfy the following relations at a surface :

$$\hat{\phi}_0^g=2\left(\overset{=g,+}{J}_0+\overset{=g,-}{J}_0\right),\ \hat{\phi}_2^g=2\left(\overset{=g,+}{J}_2+\overset{=g,-}{J}_2\right). \tag{8}$$

Eqs. (7) and (8) are consistent with internodal coupling at the node boundary. The partial moments in Eq. (6) are then equivalently expressed in terms of $\overset{=g,\pm}{J}_0$ and $\overset{=g,\pm}{J}_2$ [4]. In addition, Eq. (5) can be reformulated in the form of Eq. (1). With Eqs. (5), (7), and (8), the SP_3 equations can then be solved by the AFEN method.

3. Numerical Results

To verify the AFEN method and COREDAX code with SP_3, we solved two problems. The small LWR core benchmark problem [5] was tested and the results are shown in Table I. The IAEA3D benchmark problem (modified version for transport calculation) [6] was tested and the results are shown in Table II. The AFEN method results with diffusion and SP_3 are compared with the results of DANTSYS that is a discrete ordinates transport code..

Table I. Results of the small LWR core benchmark problem (rod-out case)

	k_{eff}	diff. (pcm)	CPU time
DANTSYS[a]	0.97676	reference	188 (sec)
COREDAX (Diffusion[b])	0.92916	4760	3 (sec)
COREDAX (SP_3[b])	0.95646	2030	10 (sec)

[a] mesh size : 0.33 cm; 4 angles; DSA acceleration
[b] mesh size : 5cm; no acceleration

Table II. Results of IAEA3D benchmark problem

	k_{eff}	diff. (pcm)	CPU time
DANTSYS[a]	1.02956	reference	2731(sec)
COREDAX (Diffusion[b])	1.02908	48	3 (sec)
COREDAX (SP_3[b])	1.02958	-2	9 (sec)

	Maximum Power Error (%)	RMS of Power Errors (%)
DANTSYS[a]	Ref.	reference
COREDAX (Diffusion[b])	2.88	1.36
COREDAX (SP_3[b])	1.35	0.46

[a] mesh size : 2.0 cm; 16 angles; DSA acceleration
[b] mesh size : 20.0 cm; CGR acceleration

4. Conclusions

The AFEN methodology has been successfully extended to the SP_3 model of a transport approximation. The test results show that SP_3 provides much improved k_{eff} and power distributions than P_1 (diffusion) in reference to the transport solutions. The increase in computing time is marginal and affordable.

REFERENCES

[1] P. S. Brantley and E. W. Larsen, The Simplified P_3 Approximation, *Nucl. Sci. Eng.*, Vol.134, p.1, 2000.
[2] Nam Zin Cho, Jaejun Lee, Do Sam Kim, Chae Yong Yang, and Jong Chull Jo, Improved Features and Verification of an AFEN Method Code in Hexagonal-Z 3-D Geometry for Neutron Diffusion Calculation, *Trans. Am. Nucl. Soc.*, Vol.92, p.514, 2005.
[3] Sweng Woong Woo and Nam Zin Cho, The Analytic Function Expansion Nodal Method Refined with Transverse Gradient Basis Functions and Interface Flux Moments, *Nucl. Sci. Eng.*, Vol.139, p.156, 2001.
[4] Bumhee Cho, Nodal Method for SP_3 Approximation to Transport Equations, MS Thesis, Korea Advanced Institute of Science and Technology (in progress).
[5] Toshikazu Takeda and Hideaki Ikeda, 3-D Neutron Transport Benchmarks, *J. Nucl. Sci. Technol.*, Vol.28, p.656, 1991.
[6] Alain Hébert, Mixed-dual implementations of the simplified P_n method, *Ann. Nucl. Energy*, Vol.37, p.498, 2010.