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A Brief Review on the Baer-Nunziato type Multi-pressure Multi-fluid Models

Sang Yong Lee*, Chan Eok Park, and Gyu Cheon Lee
Korea Power Engineering Co., 150 Deokjin-dong, Yuseong-gu, Daejeon, 305-353
**Corresponding author:sanglee@kopec.co.kr*

1. Introduction

Single pressure two-fluid flow equations have complex characteristics [16,19,20]. This causes ill-posedness problem. Even though some authors [19,20] show that the numerical solutions are well behaved if the number of mesh points is sufficiently small, the stability of the solution is always challenged.

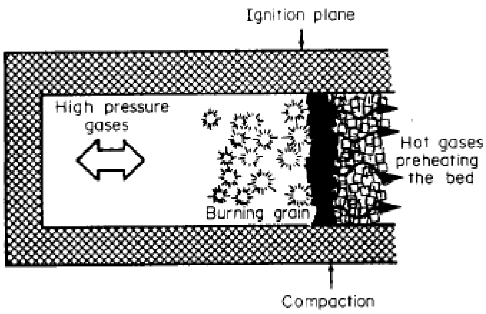

Fig.1. A schematic of flame spread in a highly confined column of explosive [2]

Table-1. Some Combinations of V_i and P_i

References	Interfacial velocity	Interfacial pressure
Baer[1] Gavrilyuk[8]	$V_i = u_1$ $V_i = u_2$	$P_i = P_2$ $P_i = P_1$
Glimm[9]	$V_i = \alpha_2 u_1 + \alpha_1 u_2$	$P_i = \alpha_2 P_1 + \alpha_1 P_2$
Coquel, Gallouet[4,9]	$V_i = u_1$ $V_i = u_2$ $V_i = \frac{\alpha_1\rho_1 u_1 + \alpha_2\rho_2 u_2}{\alpha_1\rho_1 + \alpha_2\rho_2}$	$P_i = P_2$ $P_i = P_1$ $P_i = \frac{\alpha_1\rho_1 T_1 P_2 + \alpha_2\rho_2 T_2 P_1}{\alpha_1\rho_1 T_1 + \alpha_2\rho_2 T_2}$
Saurel[13]	$V_i = \frac{\alpha_1\rho_1 u_1 + \alpha_2\rho_2 u_2}{\alpha_1\rho_1 + \alpha_2\rho_2}$	$P_i = \alpha_1 P_1 + \alpha_2 P_2$
Lhuiller[13]	$V_i = \frac{1}{2}u_1 + \frac{1}{2}u_2$	$P_i = \alpha_2 P_1 + \alpha_1 P_2$
Ransom[12]	$V_i = \frac{1}{2}u_1 + \frac{1}{2}u_2$	$P_i = \frac{1}{2}P_1 + \frac{1}{2}P_2$

There have been several attempts to overcome these problems [6]. Multi-pressure multi-fluid models are one of them [2, 12]. Among them, Baer and Nunziato (BN) derived an interesting two-fluid model. BN model has independent phase pressures. It is closed by inserting volume fraction evolution equation [2]. In this paper, several aspects of the BN type model will be reviewed and some suggestion for the future study will be made.

2. BN model

Originally, BN model is developed to be applied to the deflagration to detonation transition (DDT) phenomena in which discontinuous solutions, such as shocks and contact discontinuity, prevail. They occur when highly explosive solid grains are packed and reacted in a confinement (Fig.1) [2,3]. The balance equations are as follows;

$$\partial_t(\alpha_a) + V_i\partial_x(\alpha_a) = K_P(P_1 - P_2) \tag{1}$$

$$\partial_t(\alpha_k\rho_k) + \partial_x(\alpha_k\rho_k u_k) = \dot{m}_k \tag{2}$$

$$\begin{aligned}\partial_t(\alpha_k\rho_k u_k) + \partial_x(\alpha_k\rho_k u_k{}^2 + \alpha_k P_k) - P_i\partial_x(\alpha_k) = \\ -K_{uk}(u_1 - u_2) + \dot{m}_k V_i\end{aligned} \tag{3}$$

$$\begin{aligned}\partial_t(\alpha_k E_k) + \partial_x(\alpha_k u_k(E_k + P_k)) - V_i P_i\partial_x(\alpha_k) = \\ -K_{uk}V_i(u_1 - u_2) + \dot{m}_k E_i\end{aligned} \tag{4}$$

$$\alpha_1 + \alpha_2 = 1 \tag{5}$$

$$E_k = \rho_k e_k + \rho_k (u_k)^2 / 2 \tag{6}$$

$$e_k = e_k(P_k, \rho_k), \quad k = 1, 2 \tag{7}$$

where we note α volume fraction, ρ density, u velocity, P_c compaction pressure, V_i interface velocity, P_i interface pressure, K_P and K_u the positive functions of velocity and pressure relaxation, $\dot{m}$ the mass transfer and E_i the interfacial energy. Note that terms K_{uk} $(u_1 - u_2)$ correspond to drag force effects.

Most conspicuous aspect of the BN model is the volume fraction evolution equation (1). There have been strong debates concerning the validity of this equation. For example, Dinh [6] argues that it might not be valid with the respect of steady state argument.

Closure relations for BN type models are also controversial. Various attempts are shown in Table-1. Original suggestion is to take solid phase velocity and gas pressure P_2 as the interface velocity V_i and the interface pressure respectively. If one of phases is solid, like the original BN model, compaction pressure can be introduced [2,6]. These values have to be determined so that the entropy inequality can be met [2,9].

3. Characteristics of BN model

The characteristic analysis of the BN model is well documented by Embid [7]. The BN model is hyperbolic unlike the conventional single pressure two-fluid model. Eigenvalues of BN model are $\lambda_1^- = u_1 - c_1$, $\lambda_1^+ = u_1 + c_1$, $\lambda_2^- = u_2 - c_2$, $\lambda_2^+ = u_2 + c_2$, $\lambda_1^0 = u_1$, $\lambda_2^0 = u_2$, $\lambda^c = V_i$. Fields associated with $\lambda_1^-, \lambda_1^+, \lambda_2^-, \lambda_2^+$ are genuinely non-linear while the fields associated with the waves λ_1^0 and λ_2^0 are linearly degenerate. Important items for the characteristic analysis such as, right and left eigenvectors, generalized invariants, are found in Embid [7]. Authors [1,5,18] solved the exact Riemann problem for BN model. Recently Deledicque [5] showed that the Riemann problem can be classified into four principal families of configurations depending on the relative location of contact discontinuities of individual phases. Also he showed that it can be ill-posed for some initial conditions, i.e. the Riemann problem might possess two solutions or no solution at all.

4. Interesting Numerical calculation

Even though some authors used central differencing method [2,15], most of authors [1,9,12,17] used various approximate Riemann solvers to adopt Godunov [11] type upwind scheme to solve the BN type two-fluid model. It is generally believed that the density based upwind method may be not applicable to low Mach number flow. But Gallouet [9] showed that the model is able to work also on incompressible flows such as water faucet problem. The water faucet problem consists of a vertical tube 12 m in length. The top has a fixed liquid velocity (10 m/s) and a liquid volume fraction of 0.8. The bottom of the tube is open to atmospheric conditions. The numerical solution shows

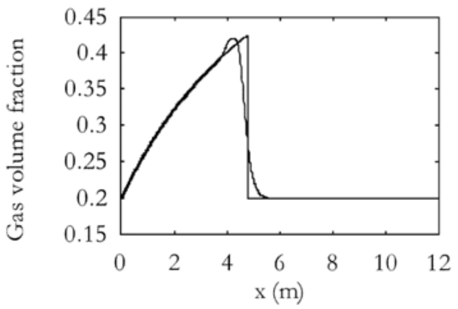

Fig.2 Gas volume fraction at time 0.4 s. (exact solution; bold lines).

good agreement with the exact one (Fig.2).

5. Conclusion and Future works

BN model has been developed mainly for the application to DDT process which has the discontinuous solutions of high Mach number flow. But several authors [1,9,12,17] tried to extend its applicability to gas water system. Guillemaud [12] even simulated the case with phase transition during the depressurization. Herard [13] extends the BN model to three-fluid with relevant closure relations.

Now, there are two suggestions for the future study. One is to apply the conventional density based upwind method to more extensive low Mach number flow with various source terms concerning mass, momentum and energy exchange effects. The other one is to develop the conventional pressure based scheme, such as ICE [14] method, for continuous solutions. Since individual phase pressures are dependent variables, two independent system pressure equations have to be constructed. There may be two methods to treat the volume fraction evolution equation. It can be solved implicitly with old convection velocity. Or, it can be merged with other conservation equations by using implicit convection velocity.

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