# Decomposition of Resonance Fixed Source Equation for MOC Calculation

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## 1. Introduction

Resonance fixed source problem is a boundary condition problem. Therefore, if the incoming angular fluxes are given at the problem boundary, this problem can be solved by one ray sweeping in MOC calculation. Also Resonance fixed source problem can be efficiently solved by decomposing the original problem into homogeneous and heterogeneous problems. In this paper, these two concepts are introduced on the resonance fixed source equation for the MOC codes such as DeCART [1] and KARMA [2].

## 2. Methods and Results

The resonance fixed source equation can be written as:

$$\Omega \cdot \nabla \varphi + \left(\Sigma_{am} + \lambda\Sigma_p\right)\varphi = \lambda\Sigma_p \,, \qquad (1)$$

where, $\Sigma_{am}$ is determined by the *m*-th subgroup level cross section and $\lambda\Sigma_p$ by the intermediate potential cross section. The equivalent cross section is obtained from the solution of scalar flux of Eq. (1), and then it is tabularized as a function of subgroup level cross sections. The tabularized equivalent cross section is used in subgroup method to generate the resonance cross section.

In Eq. (1), the neutron source and cross sections are all fixed, and the solution is only dependent on the boundary angular fluxes. Therefore, the computational performance in solving Eq. (1) is determined by: (1) how to calculate the boundary angular fluxes, and (2) how to solve Eq. (1) efficiently. In this chapter, a back tracing and decomposition concepts are introduced to resolve these 2 questions.

### *2.1 Back Tracing Concept*

The boundary angular flux can be determined by accumulating the regional source contribution as:

$$\varphi = \sum_{i=1}^{N} S_i E_i T_i \,, \qquad (2)$$

where

$S_i$ = neutron source of region *i*,

$E_i = 1 - e^{\tau_i}$ = escape probability,

$T_i = \prod_{j=1}^{i-1} e^{\tau_j}$ = transmission probability,

$\tau_i$ = optical length.

The contribution probability of regional source to the boundary angular flux can be calculated by multiplying the escape and the transmission probabilities.

The old DeCART performs a few number of ray sweeping iterations for the resonance fixed source problem to get a converged scalar flux and boundary angular flux as the normal ray sweeping in the eigenvalue calculations. In the back tracing concept, the boundary angular flux is calculated by using Eq. (2). The regional sources whose contribution probabilities are less than 10-5 are ignored.

### *2.2 Decomposition Scheme*

In the decomposition concept, the original resonance fixed source equation is decomposed into the homogeneous and heterogeneous equations. In Eq. (1), the source term of $\lambda\Sigma_p$ appears in most of all regions but $\Sigma_{am}$ in a few resonance regions. If we decompose the angular flux into homogeneous and heterogeneous solutions as:

$$\varphi = \varphi_0 + \varphi_1 \,, \qquad (3)$$

where $\varphi_0$ is the solution of the following homogeneous equation as :

$$\Omega \cdot \nabla \varphi_0 + \lambda\Sigma_p \varphi_0 = \lambda\Sigma_p \,. \qquad (4)$$

Then the original equation can be reduced to the following heterogeneous equation as:

$$\Omega \cdot \nabla \varphi_1 + \left(\Sigma_{am} + \lambda\Sigma_p\right)\varphi_1 = -\Sigma_{am}\varphi_0 \,. \qquad (5)$$

In the decomposition scheme, the homogeneous and heterogeneous equations are solved instead of the original equation. The solution of homogeneous equation is known to be '1.0' for the reflective boundary condition. Therefore, the homogeneous equation is solved only for the non-reflective boundary problem. Also, the solution of homogeneous equation is independent on the subgroup flux level or the resonance category because $\lambda\Sigma_p$ is fixed by the group-wise isotope cross sections. Therefore, the homogeneous equation is solved only once for a resonance energy group if a non-reflecting boundary condition is assigned.

Fig. 1 shows the heterogeneous problem comparing with the original problem. The heterogeneous equation is easier to solve than the original equation because the given fixed sources are zero for the non-resonance regions. If the zero sources appear in several subsequent

regions, those zero source regions can be passed in one time by just accounting for the total attenuation factor for the angular flux.

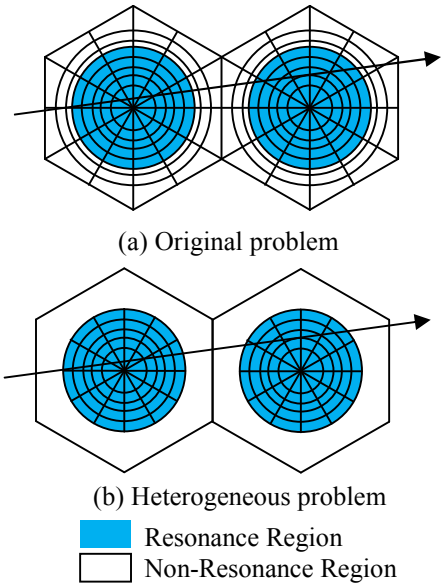


Fig. 1 Heterogeneous Problem

*2.3 Results*

The computational efficiencies of the above concepts are examined for hexagonal single pin, assembly and core problems. In this examination, the 190-G neutron library which contains 69 resonance groups and 4 sub-group levels is used. The single pin and assembly problems are solved by using 0.02 cm ray spacing, 8 azimuthal angles per sextant and 3 polar angles. The core problem which consists of 54 fuel blocks and 73 reflector blocks is solved with 1/12 symmetric condition and by using 0.05 cm ray spacing, 4 azimuthal angles per sextant and 2 polar angles.

Table 1 shows the computational efficiency improvement on resonance fixed source calculations by introducing the back tracing and the decomposition concepts. In single pin and assembly problems which use reflective boundary condition, the back tracing concept shows about 1.60 and 1.84 speedups compared with the original boundary angular flux iteration scheme. In the core problem that uses the vacuum boundary condition, the back tracing concept is not incorporated in the calculation due to the zero incoming angular flux. The decomposition scheme shows about 1.6 speedups for all the problems. While the homogeneous equation is not solved in the single pin and assembly problems due to the known solution of 1.0 for all flat source regions, it is solved in the core problem. Even though the core problem solves the homogeneous equation, the speedup of decomposition scheme is compatible with other problems, which is mainly due to the much more number of non-resonance regions in the reflector blocks. Introduction of both the back tracing and the decomposition concepts shows the speedups of about 2.7 for reflective boundary problems and about 1.6 for vacuum boundary problems.

Table 1: Computing Time on Resonance Fixed Source Calculation (Intel(R) Core(TM) i7 2.67 GHz)

| Scheme | Time (Speedup)* | | |
|---|---|---|---|
| | Pin | Assembly | Core |
| Boundary Iteration | 5.23 (1.00) | 139.8 (1.00) | 783 (1.00) |
| Only Back Tracing | 3.26 (1.60) | 75.9 (1.84) | 783 (1.00) |
| Back Tracing + Decomposition | 1.97 (2.65) | 49.4 (2.83) | 495 (1.58) |

*) Total 69 Resonance Groups, 2 Representative Categories, 4 Subgroup Levels in 190-G Library

## 3. Conclusions

In this paper, the back tracing and the decomposition concepts were introduced to the resonance fixed source equations for the MOC codes such as DeCART and KARMA. The back tracing concept was available for the pin and the assembly problems using the reflective boundary condition and it showed about 1.7 speedups. The decomposition scheme was useful for all the problems and it showed about 1.6 speedups. Introduction of both the back tracing and the decomposition concepts showed the speedups of about 2.7 for reflective boundary problems and about 1.6 for vacuum boundary problems. Therefore it was concluded that the back tracing and the decomposition concepts could be successfully applicable to the MOC codes and it could improve the computational efficiency on the resonance fixed source calculation maximum about 2.7 times.

## ACKNOWLEDGMENTS

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## REFERENCES

[1] J-Y. Cho, K-S. Kim, H-J. Shim, J-S. Song, C-C. Lee and H-G. Joo, Whole Core Transport Calculation Employing Hexagonal Modular Ray Tracing and CMFD Formulation, Journal of Nuclear Science and Technology, 45, no. 8, 740-751 (2008).
[2] K-S. Kim, S-G. Hong, J-Y. Cho, and J-S. Song, "Transport Lattice Code KARAM 1.1," Transactions of the KNS Autumn Meeting, Gyeongju, Oct. 29-30, 2009.