Cellular Neural Network Method for Critical Slab with Albedo Boundary Condition

A. Pirouzmand[a,b*], K. Hadad[a], and K.Y. Suh[b]
[a]*Department of Nuclear Engineering, Shiraz University, Shiraz, Iran*
[b]*Department of Energy Systems Engineering, Seoul National University*
599 Gwanak-Ro, Gwanak-Gu, Seoul, 151-744, Korea
[*]*Corresponding author: pirouzm@shirazu.ac.ir*

1. Introduction

The neutron transport problems have been studied theoretically and numerically for years. A number of researchers have studied the criticality problems of one-speed neutrons in homogeneous slabs and spheres using various methods [1-2]. The Chebyshev polynomial approximation method (T_N method) has lately been developed and improved for the neutron transport equation in slab geometry [3]. The one-speed time-dependent neutron transport equation using the Cellular Neural Network (CNN) for the vacuum boundary condition has previously been solved [4]. In this paper, we demonstrate the capacity of CNN in calculating the critical slab thickness for different boundary conditions and its variation with moments N. The architecture of the CNN has already been dealt with thoroughly [4]. Essentially, the CNN is used to model a first-order system of the partial differential equations (PDEs). The original equations in the T_N approximation are also a set of PDEs. The CNN approach lends itself to analog VLSI implementation. In this study, the CNN model is implemented using the HSpice software package.

2. T_N Approximation

With isotropic scattering and no sources in slab geometry, the single-energy time-dependent Boltzmann integro-differential equation is

$$\frac{1}{v}\frac{d\psi(x,\mu,t)}{dt}+\mu\frac{d\psi(x,\mu,t)}{dx}+\sigma_T\psi(x,\mu,t)=\frac{\sigma_s}{2}\int_{-1}^{1}\psi(x,\mu',t)d\mu' \qquad -a\le x\le a,\quad -1\le\mu\le 1, \tag{1}$$

where μ is cosine of the angle; $\psi(x,\mu)$ is angular flux; v is velocity; t is time; σ_T and σ_S are total and scattering differential cross sections, respectively.

By using Chebyshev polynomials of the first kind expansion, the T_N moments of the angular flux are [4]

$$\begin{cases}\frac{1}{v}\frac{d\phi_o(x,t)}{dt}+\frac{d\phi_1(x,t)}{dx}+\sigma_T\phi_o(x,t)=\sigma_S\phi_o(x,t), & n=0\\ \frac{1}{v}\frac{d\phi_1(x,t)}{dt}+\frac{1}{2}\frac{d\phi_2(x,t)}{dx}+\frac{1}{2}\frac{d\phi_o(x,t)}{dx}+\sigma_T\phi_1(x,t)=0, & n=1\\ \frac{1}{v}\frac{d\phi_n(x,t)}{dt}+\frac{1}{2}\frac{d\phi_{n+1}(x,t)}{dx}+\frac{1}{2}\frac{d\phi_{n-1}(x,t)}{dx} & \\ +\sigma_T\phi_n(x,t)=\frac{1}{2}\frac{[1+(-1)^n]}{1-n^2}\sigma_S\phi_o(x,t), & n\ge 2\end{cases} \tag{2}$$

The Albedo boundary is

$$\psi(a,-\mu,t)=R\psi(a,\mu,t),\quad \mu>0 \tag{3}$$

Applying the Marshak type boundary condition to Eq. (3) results in

$$\int_0^1\left(\psi(a,-\mu)-R\psi(a,\mu)\right)T_k(-\mu)d\mu=0,\qquad k=1,3,5,...,N \tag{4}$$

A similar expression to Eq. (4) can be obtained for slab's left side (x=-a).

3. CNN Model for T_N Approximation

A multilayer CNN in a discretized x-coordinate is applied. The moments of neutron flux at each grid point are $\phi_{o,i},\phi_{1,i},......,\phi_{n,i}$ associated with $\phi_o(i\Delta x),\phi_1(i\Delta x),......,\phi_n(i\Delta x)$ for $i=0, 1, 2,..., L$, where L is the number of nodes in the x direction.

Upon truncated Taylor series expansion in space, Eq. (2) becomes

$$\begin{cases}\frac{1}{v}\frac{d\phi_{o,i}(t)}{dt}=\frac{\phi_{1,i-1}-\phi_{1,i+1}}{2\Delta x}-(\sigma_T-\sigma_S)\phi_{o,i}\\ \frac{1}{v}\frac{d\phi_{1,i}(t)}{dt}=\frac{1}{2}\frac{\phi_{2,i-1}-\phi_{2,i+1}}{2\Delta x}+\frac{1}{2}\frac{\phi_{o,i-1}-\phi_{o,i+1}}{2\Delta x}-\sigma_T\phi_{1,i}\\ \frac{1}{v}\frac{d\phi_{n,i}(t)}{dt}=\frac{1}{2}\frac{\phi_{n+1,i-1}-\phi_{n+1,i+1}}{2\Delta x}+\frac{1}{2}\frac{\phi_{n-1,i-1}-\phi_{n-1,i+1}}{2\Delta x}\\ \qquad -\sigma_T\phi_{n,i}+\frac{1}{2}\frac{[1+(-1)^n]}{1-n^2}\sigma_S\phi_{o,i},\quad n\ge 2\end{cases} \tag{5}$$

Comparison of Eq. (5) with the CNN state equation [4] results in a one-dimensional (N+1) layer CNN with the following cell elements

$$\begin{cases}R_x^{(\phi_o)}=1/(\sigma_T-\sigma_S)\ , \qquad A^{(\phi_o\phi_1)}(i,i-1)=\frac{1}{2\Delta x}\\ A^{(\phi_o\phi_1)}(i,i+1)=-\frac{1}{2\Delta x}\\ A^{(\phi_1\phi_0)}(i,i-1)=A^{(\phi_1\phi_2)}(i,i-1)=\frac{1}{4\Delta x}\\ A^{(\phi_1\phi_0)}(i,i+1)=A^{(\phi_1\phi_2)}(i,i+1)=-\frac{1}{4\Delta x}\end{cases} \tag{6a}$$

$$\begin{cases}C^{(\phi_k)}=\frac{1}{v}, & k=0,1,2,...,N\\ R_x^{(\phi_k)}=1/\sigma_T, & k=1,2,...,N\end{cases} \tag{6b}$$

$$\begin{cases} A^{(\phi_k\phi_{k-1})}(i,i-1) = A^{(\phi_k\phi_{k+1})}(i,i-1) \\ \qquad = \dfrac{1}{4\Delta x}, \ k = 2, 3, ..., N \\ A^{(\phi_k\phi_{k-1})}(i,i+1) = A^{(\phi_k\phi_{k+1})}(i,i+1) \\ \qquad = -\dfrac{1}{4\Delta x}, \ k = 2, 3, ..., N \\ A^{(\phi_k\phi_0)}(i,i) = \sigma_S \dfrac{1}{2}\dfrac{[1+(-1)^k]}{1-k^2}, \ k = 2, 3, ..., N \end{cases} \tag{6c}$$

where $C^{(k)}$ and $R_x^{(k)}$ are the k^{th} layer capacitance and resistance, and $A^{(nn')}(i,i')$ is the linear voltage controlled current source gain which couples node i in the n^{th} layer with i' in the n'^{th} layer.

4. Results

The CNN model is programmed into HSpice. A total of 21 grids is considered in the x direction. The CNN elements are evaluated by Eq. (6). The CNN, while having a time dependent dynamic, can be used for finding the critical slab thickness when the voltages of all nodes in the circuit reach a stable state. This goal can be achieved by selecting an initial value for Δx in the CNN model for each value of R, N and c where $c = \sigma_S/\sigma_T$ is the mean number of secondary neutrons per collision and R is the reflection coefficient, and then changing the value of Δx to reach the stable state. The computed critical half thicknesses for vacuum boundary condition (R=0) with various values of N and C are given in Table I. Tables II through V list the computed critical half thicknesses for R=0.25, 0.5, 0.75 and 0.99, respectively.

Table I. Computed Critical Half Thickness (cm) for R=0

c	T_5	T_7	T_9
1.01	8.333107	8.330770	8.330480
1.1	2.117674	2.114707	2.114380
1.3	0.944786	0.939611	0.939044
1.6	0.524227	0.515856	0.514031
2.0	0.327775	0.318040	0.314830

Table II: Computed Critical Half Thickness (cm) for R=0.25

c	T_5	T_7	T_9
1.01	7.89337	7.88951	7.88915
1.1	1.77576	1.77256	1.77150
1.6	0.37426	0.37655	0.36851
2.0	0.22413	0.21646	0.21527

Table III: Computed Critical Half Thickness (cm) for R=0. 5

c	T_5	T_7	T_9
1.01	7.06623	7.06105	7.06007
1.1	1.28828	1.28490	1.28453
1.6	0.22922	0.22587	0.22510
2.0	0.13305	0.12985	0.12873

Table IV: Computed Critical Half Thickness (cm) for R=0. 75

c	T_5	T_7	T_9
1.01	5.09028	5.08261	5.08220
1.1	0.65644	0.65431	0.65410
1.6	0.10260	0.10128	0.10103
2.0	0.05861	0.05730	0.05697

Table V: Computed Critical Half Thickness (cm) for R=0. 99

c	T_5	T_7	T_9
1.01	0.25067	0.25015	0.25030
1.1	0.02441	0.02436	0.02431
1.6	0.00365	0.00353	0.00417
2.0	0.00208	0.00208	0.00208

5. Conclusions

In this work, we examined the critical half thickness of the homogenous medium slab for one-speed and with isotropic scattering using the T_N method coupled with CNN. We computed the critical half thickness of the slab by using the Marshak boundary condition. For the various c and R values, computed results are sown to be in good agreement with the P_N results in the literature [3]. We assumed that the total differential cross section is $\sigma_T = 1cm^{-1}$. The CNN model is an online and real-time system. It is thus possible to simulate the effect of system parameter variations (e.g. c and Δx) on the total flux while the simulation is in progress. The advantage of the CNN simulator over the numerical methods has to do with its analog and parallel processing algorithm. One can find the best value for Δx very quickly.

REFERENCES

[1] D.C. Sahni, N.G. Sjöstrand," Criticality and time eigenvalues in one-speed neutron transport," Progress in Nuclear Energy, Vol. 23, p. 241, 1990.
[2] C. Yıldız, "Variation of the critical slab thickness with the degree of strongly anisotropic scattering in one-speed neutron transport theory," Annals of Nuclear Energy, Vol. 25, p. 529, 1998.
[3] F. Anli, F. Yaşa, S. Güngör, H. Öztürk, "T_N approximation to neutron transport equation and application to critical slab problem," Quantitative Spectroscopy and Radiative Transfer, Volume 101, Issue 1, pp. 129-134, September 2006.
[4] K. Hadad, A. Pirouzmand, N. Ayoobian, "Cellular neural networks (CNN) simulation for the T_N approximation of the time dependent neutron transport equation in slab geometry," Annals of Nuclear Energy, Vol. 35, pp. 2313-2320, 2008.