# Implementations of Three Models to Affect Strongly on Lateral Void Distribution Estimation in the CUPID

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## 1. Introduction

The void distribution for the lateral direction to the primary flow motion is important for the general flow structure in the case of the pipe flow. The turbulence model, the non-drag model, and the interfacial area concentration model are known as the important models related to this lateral void distribution.

Turbulence models are directly related to the lateral direction velocity profiles. The zero equation and k-ε turbulence models based upon the eddy viscosity are used for calculating the turbulence viscosity. The wall function is adopted for the wall boundary condition of the turbulence viscosity. While the standard drag force and the virtual mass force are related to the relative movement to the primary flow motion, the non-drag forces are related to the vapor movement to the lateral direction of the primary flow motion. The interfacial area concentration transportation equation is set up to calculate the interfacial area concentration distribution. This transport equation is strongly affected by the turbulence model and the non-drag forces.

In this paper, the implementations into the CUPID [1] of the turbulence model, non-drag force, and the interfacial concentration transport equation are summarized.

## 2. Mathematical Model

### *2.1 Governing Equation*

The governing equations of the two-fluid, three-field model are similar to those of the time-averaged two-fluid model derived by Ishii and Hibiki [2]. The momentum equation for the k-phase is given by

$$\frac{\partial}{\partial t}(\alpha_k \rho_k \underline{u}_k) + \nabla\cdot(\alpha_k \rho_k \underline{u}_k \underline{u}_k) = -\alpha_k \nabla P + \nabla\cdot[\alpha_k(\mu+\mu_T)\nabla\vec{u}_k] + \alpha_k \rho_k \underline{g} + P\nabla\alpha_k + M_k^{mass} + M_k^{drag} + M_k^{VM} + +M_k^{non-drag} \quad (1)$$

where $\alpha_k, \rho_k, u_k, P$ are the k-phase volume fraction, density, velocity, pressure. $M_k$ represents the interfacial momentum transfer due to a mass exchange, a standard drag force, and several drag forces except the standard drag force virtual mass. $M_k^{non-drag}$ includes the virtual mass force, the lift force, the wall lubrication force, and turbulence dispersion force.

$$M_k^{non-drag} = M_k^L + M_k^{WL} + M_k^{TD} \cdot \quad (2)$$

### *2.2 Turbulence Model*

A very simple eddy viscosity model, called as a zero equation model, computes a global value for $\mu_T$ from the mean velocity and a geometric length scale using an empirical formula.

$$\mu_T = \rho C_\mu u_T l_T \cdot \quad (3)$$

where $C_\mu$ and $l_T$ are a proportionality constant and turbulence length scale. For the physically meaningful consideration of the turbulence effect, k-ε turbulence model was also implemented for liquid phase.

$$\mu_{T,l} = C_\mu \rho_l \frac{k_l^2}{\varepsilon_l}, \quad (4)$$

$$\frac{\partial(\alpha_l \rho_l k_l)}{\partial t} + \nabla\cdot(\alpha_l \rho_l k_l \vec{u}_l) = \nabla\cdot\left[\alpha_l\left(\mu + \frac{\mu_T}{\sigma_k}\right)_l \nabla k\right] + \alpha_l P_k - \alpha_l \rho_l \varepsilon, \quad (5)$$

$$\frac{\partial(\alpha_l \rho_l \varepsilon_l)}{\partial t} + \nabla\cdot(\alpha_l \rho_l \varepsilon_l \vec{u}_l) = \nabla\cdot\left[\alpha_l\left(\mu + \frac{\mu_t}{\sigma_k}\right)_l \nabla\varepsilon\right] + \frac{\alpha_l \varepsilon}{k}(C_{\varepsilon 1} P_k - C_{\varepsilon 2} \rho_l \varepsilon). \quad (6)$$

The effective viscosity of the continuous liquid phase is the sum of the laminar viscosity, the turbulence viscosity, and the bubble effect. The effective viscosity of the dispersed gas phase can be calculated by assuming the same kinematic viscosity of the liquid and gas.

$$\mu_{T,g} = C_{\mu,g} \rho_l \alpha_g d_g \left|\vec{U}_g - \vec{U}_l\right|. \quad (7)$$

The wall-function is an extension of the method of Launder and Spalding[3]. The logarithmic relation for the near wall velocity is given by:

$$u^+ = \frac{U_t}{u_\tau} = \frac{1}{\kappa}\ln(Cy^+) = \frac{1}{\kappa}\ln\left(C\frac{\rho \Delta y u_\tau}{\mu}\right). \quad (8)$$

### *2.3 Non-drag Forces*

The void distribution in the near wall region is important for the general flow structure in the case of the pipe flow. It mainly determined by the lift and the wall forces. The lift force [3] pushes the bubble with perpendicular to the liquid motion. The lift force is given in terms of the slip velocity and the curl of the continuous phase velocity by:

$$M_l^L = -\alpha_g \rho_l C_L (\vec{u}_g - \vec{u}_l) \otimes (\vec{\nabla} \otimes \vec{u}_l). \quad (9)$$

Here $C_L$ is 0.5 for an inviscid flow around a sphere, but it can be between 0.01 and 0.05 for a viscous flow. The wall lubrication correlation like that by Antal et al. [4] are tested as

$$M_l^{WL} = \frac{-\alpha_g \rho_l \left|\vec{u}_g - \vec{u}_l\right|^2}{d} \max\left(0, C_1 + C_2 \frac{d_{bubble}}{y_{wall}}\right)\vec{n} \quad (10)$$

with $C_1$ =-0.01, $C_2$ =0.05. The wall lubrication force is limited within 5 particle diameters from the wall. This force can be seen on fine grids by considering the bubble diameter.

Considering the turbulence dispersion force by Bertadano, Burns et al. [5] suggested the model for the turbulence dispersion force as following:

$$M_l^{TD} = -C_{TD} C_D \frac{\nu_{T,g}}{Sc_{T,g}} \left( \frac{\nabla \alpha_l}{\alpha_l} - \frac{\nabla \alpha_g}{\alpha_g} \right) \quad (11)$$

where $C_{TD}, C_D, \nu_{T,g}, Sc_{T,g}$ indicate turbulence dispersion coefficient(~1), the drag coefficient(~2), the turbulence kinematic viscosity for gas, turbulent Schumidt number or turbulent Prandtl number(~0.9). The non-drag forces for the gas phase have the same magnitude and an opposite sign as follows.

$$M_g^{non-drag} = -M_l^{non-drag} \quad . \quad (12)$$

*2.4 Interfacial Area Transport Equation*

For a multi-dimensional calculation of the IAC (interfacial area concentration), Yao and Morel[6] derived an interfacial area transport equation available for a boiling flow as follows.

$$\frac{\partial a_i}{\partial t} + \nabla \cdot (a_i V_g) = \frac{2}{3} \frac{a_i}{\alpha_g \rho_g} \left[ \Gamma_{i,g} - \alpha_g \frac{d\rho_g}{dt} \right] + \phi_{CO} + \phi_{BK} + \phi_{PH} \quad (13)$$

where $\phi_{CO}, \phi_{BK}, \phi_{PH}$ mean the variance of IAC by a coalescence, breakup and nucleation, respectively. The first term on the right-hand side of the equation is the term of a bubble size variance due to a condensation heat transfer or a pressure drop. Considering a bubbly flow region, the coalescence by a random collision (RC) and the breakup by a turbulent impact (TI) are considered for the second and the third terms on the right-hand side of Eq (13), respectively.

$$\phi_{RC} = \frac{1}{3\psi} \left( \frac{\alpha_v}{a_i} \right)^2 K_{c1} \frac{\varepsilon^{1/3} \alpha_v^2}{D_{sm}^{11/3}} \frac{1}{g(\alpha_v) + K_{c2} \alpha \sqrt{We/We_c}} \exp\left( -K_{c3} \sqrt{\frac{We}{We_c}} \right) , \quad (14)$$

$$\phi_{TI} = \frac{1}{3\psi} \left( \frac{\alpha_v}{a_i} \right)^2 K_{b1} \frac{\varepsilon^{1/3} \alpha_v (1-\alpha_v)}{D_{sm}^{11/3}} \frac{1}{1 + K_{b2} (1-\alpha) \sqrt{We/We}} \exp\left( -\frac{We}{We_c} \right) , \quad (15)$$

$$\phi_{ph} = \pi \left\{ 1.5 \cdot 10^{-4} \sqrt{\frac{\sigma}{g \Delta \rho}} \left[ \frac{\rho_l c_{pl} (T_{wall} - T_{sat})}{\rho_g h_{lg}} \right]^{5/4} \right\}^2 \frac{N'' f A_H}{V_{cell}} \quad . \quad (16)$$

## 3. Qualitative Verification

The three non-drag forces of lift, turbulence dispersion, and wall lubrication, vapor volume fraction and Sauter mean diameter of bubbles, and the interfacial area concentration distributions are presented in Figure 1 and Figure 2. The profiles of the non-drag forces, bubble diameters, vapor fraction, and interfacial area concentration indicate that the turbulence model, the non-drag force models, and the interfacial area transport equation are properly implemented and work well.

## 4. Conclusions

This paper summarizes the implementation into the CUPID of turbulence model, non-drag forces, and interfacial area concentration transport and reports qualitative verification results. The non-drag forces model have a significant effect on the convergence and the k-ε model has a difficulty with profiled inlet condition. This problem might be revised by implicit treatment of the velocity terms in the lift force model and in diffusion terms of momentum equation.

## ACKNOWLEDGMENTS

This work was supported by Nuclear Research & Development Program of the KOSEF (Korea Science and Engineering Foundation) grant funded by the MEST (Ministry of Education, Science and Technology) of the Korean government (Grant code: M20702040002-08M0204-00210).

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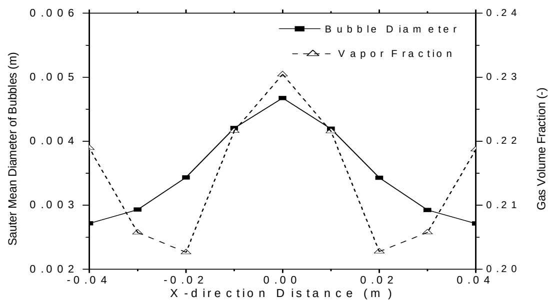


**Fig. 1 Volume Fraction and Diameter of Bubbles**

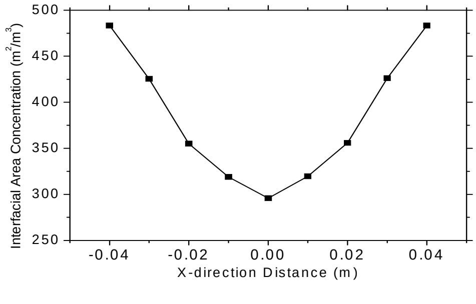


**Fig. 2 Interfacial Area Concentration Distribution**