Reliability of Multiple Redundant Systems in Consideration of Common Cause Failure

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1. Introduction

A reactor protection system (RPS) in a nuclear power plant includes multiple processing channels for ensuring both safety and economy. Lu and Lewis [1] suggested a method of unavailability and spurious operation probability (SOP) estimation for this kind of multiple redundant safety system. They treat independent failures as the main focus of study since they consider that sufficient diversities including physical and technical separation among channels are effective in circumventing common cause failures (CCF) in a CANDU-type nuclear plant.

However, most of pressurized water reactors (PWR), including OPR1000 plants in Korea, have single shutdown mechanism for an emergency, excluding some parameters covered by diverse protection system. Authors had developed some formulas which considered the CCFs as additional reason of system failures for simple a *k*-out-of-*n* system [2].

This study aims to enlarge the scope of previous study to specific voting logics (including selective 2-out-of-4) and to develop the equations to accommodate the possible combinations of independent events and CCF events.

2. Unavailability

Suppose that the n channels are identical and concurrent operation of k channels initiates the reactor trip. The system unavailability consists of three parts: The unavailability from the independent failures (U_{IND}), the unavailability from the CCF (U_{CCF}) and the unavailability from the combination of CCF and independent failures (U_{COMB}).

$$U = U_{IND} + U_{CCF} + U_{COMB} \quad (1)$$

Q_i denotes the probability that exactly i channels fail to perform given safety function. When $i \geq n-k+1$, Q_i implies the failure of trip signal generation by the RPS. $Q_{IND(i)}$ and $Q_{CCF(i)}$ implies the probability of i channels' failure caused by independent failures and the CCF respectively. U_{IND} is the probability of conditional event that a system fails due to independent failures given that there is no CCF.

$$U_{IND} = \overline{Q}_{CCF} \sum_{i=n-k+1}^{n} Q_{IND\,(i)} = \overline{Q}_{CCF} \sum_{i=n-k+1}^{n} \binom{n}{i} q^{i}(1-q)^{n-i} \quad (2)$$

where $\overline{Q}_{CCF} = 1 - \sum_{j=2}^{n} \binom{n}{j} q_{CCF\,(j)}$ and q denotes the channel failure probability.

The contribution of multiplied CCF probabilities of difference causes (e.g., $Q_{CCF(2)} \times Q_{CCF(3)}$) is much smaller than that of single cause CCF (e.g., $Q_{CCF(5)}$) for the given i=5. Thus we use single CCF event for all the causes of CCF for simplicity. With this 'no CCF multiplication (NCM) approximation',

$$U_{CCF} \approx \sum_{i=n-k+1}^{n} \binom{n}{i} q_{CCF\,(i)} \quad (3)$$

$$U_{COMB} = \sum_{i=2}^{n-k} \left[Q_{CCF\,(i)} \sum_{j=n-k+1-i}^{n-i} Q_{IND\,(j)} \right] \approx \sum_{i=2}^{n-k} \left[\binom{n}{i} q_{CCF\,(i)} \times \{ \sum_{j=n-k+1-i}^{n-i} \binom{n-i}{j} q^{j} (1-q)^{n-i-j} \} \right] \quad (4)$$

In this study, we use the Alpha Factor model for estimating the CCF probability, $q_{CCF(i)}$. That is, $\sum_{i=1}^{n} \alpha_i^{(n)} = 1$ and $\hat{\alpha}_i^{(n)} = d_i / \sum_{i=1}^{n} d_i$ where n and d_i denotes the total number of components and the number of observed failure of i components respectively.

If the system is tested by non-staggered test strategy,

$$q_{CCF\,(i)} = \frac{n}{\binom{n}{i}} \frac{\alpha_i^{(n)}}{\sum_{j=1}^{n} (j \alpha_j^{(n)})} q = \frac{n}{\binom{n}{i}} \frac{\alpha_i^{(n)}}{\alpha_t} q \quad (5)$$

where, $\alpha_t = \sum_{k=1}^{n} (k \alpha_k^{(n)})$ and $q = \sum_{i=1}^{n} \binom{n-1}{i-1} q_{CCF\,(i)}$.

From equations (3), (4) and (5),

$$U_{CCF} \approx \sum_{i=n-k+1}^{n} \binom{n}{i} \frac{n}{\binom{n}{i}} \frac{\alpha_i^{(n)}}{\alpha_t} q = \frac{nq}{\alpha_t} \sum_{i=n-k+1}^{n} \alpha_i^{(n)} \quad (6)$$

$$U_{COMB} \approx \sum_{i=2}^{n-k} \left[\binom{n}{i} \frac{n}{\binom{n}{i}} \frac{\alpha_i^{(n)}}{\alpha_t} q \{ \sum_{j=n-k+1-i}^{n-i} \binom{n-i}{j} q^{j} (1-q)^{n-i-j} \} \right] = \frac{nq}{\alpha_t} \sum_{i=2}^{n-k} \left[\alpha_i^{(n)} \{ \sum_{j=n-k+1-i}^{n-i} \binom{n-i}{j} q^{j} (1-q)^{n-i-j} \} \right] \quad (7)$$

Then,

$$U \approx \sum_{i=n-k+1}^{n} \left[\overline{Q}_{CCF} \binom{n}{i} q^{i} (1-q)^{n-i} + \frac{nq}{\alpha_t} \alpha_i^{(n)} \right] + \frac{nq}{\alpha_t} \sum_{i=2}^{n-k} \left[\alpha_i^{(n)} \{ \sum_{j=n-k+1-i}^{n-i} \binom{n-i}{j} q^{j} (1-q)^{n-i-j} \} \right] \quad (8)$$

If a system uses a specific voting logic, given i channels' failure, some of them will cause the system failure but the others will not. With identifier, C_{ij},

$$U_{IND} = \overline{Q}_{CCF} \sum_{i=1}^{n} \sum_{j=1}^{\binom{n}{i}} C_{ij} q^{i} (1-q)^{n-i} \quad (9)$$

$$U_{CCF} \approx \sum_{i=1}^{n}\sum_{j=1}^{\binom{n}{i}} C_{ij}\frac{n}{\binom{n}{i}}\frac{\alpha_i^{(n)}}{\alpha_t} q = \frac{nq}{\alpha_t}\sum_{i=1}^{n}\sum_{j=1}^{\binom{n}{i}}\frac{C_{ij}}{\binom{n}{i}}\alpha_i^{(n)} \quad (10)$$

If the *j*th configuration of *i* channels' failure results in the system function failure, C_{ij}=1, otherwise C_{ij}=0. With an additional identifier, D_{ijk},

$$U_{COMB} \approx \frac{nq}{\alpha_t}\sum_{i=1}^{n}\sum_{j=1}^{\binom{n}{i}}\left[\frac{(1-C_{ij})}{\binom{n}{i}}\alpha_i^{(n)}\{\sum_{k=1}^{n-i} D_{ijk}\, q^k (1-q)^{n-i-k}\}\right]. \quad (11)$$

If the *k*th independent failure in addition to the *j*th configuration of *i* channels CCF results in the system function failure, D_{ijk}=1, otherwise D_{ijk}=0.

3. Spurious Operation Probability

With similar derivation, for SOP,

$$S = S_{IND} + S_{CCF} + S_{COMB}\ . \quad (12)$$

$$S_{IND} = \overline{P}_{CCF}\sum_{i=k}^{n} P_{IND\,(i)} = \overline{P}_{CCF}\sum_{i=k}^{n}\binom{n}{i}p^i(1-p)^{n-i} \quad (13)$$

$$S_{CCF} \approx \sum_{i=k}^{n}\binom{n}{i}\frac{n}{\binom{n}{i}}\frac{\alpha_i^{(n)}}{\alpha_t}p = \frac{np}{\alpha_t}\sum_{i=k}^{n}\alpha_i^{(n)}\ . \quad (14)$$

$$S_{COMB} \approx \sum_{i=2}^{k-1}\left[\binom{n}{i}\frac{n}{\binom{n}{i}}\frac{\alpha_i^{(n)}}{\alpha_t}p\{\sum_{j=k-i}^{n-i}\binom{n-i}{j}p^j(1-p)^{n-i-j}\}\right] \quad (15)$$

$$= \frac{np}{\alpha_t}\sum_{i=2}^{k-1}\left[\alpha_i^{(n)}\{\sum_{j=k-i}^{n-i}\binom{n-i}{j}p^j(1-p)^{n-i-j}\}\right]$$

Then,

$$S \approx \sum_{i=k}^{n}\left[\overline{P}_{CCF}\binom{n}{i}p^i(1-p)^{n-i}+\frac{np}{\alpha_t}\alpha_i^{(n)}\right]+\frac{np}{\alpha_t}\sum_{i=2}^{k-1}\left[\alpha_i^{(n)}\{\sum_{j=k-i}^{n-i}\binom{n-i}{j}p^j(1-p)^{n-i-j}\}\right] \quad (16)$$

For a specific voting logic, with the same identifiers as in unavailability calculation, C_{ij} and D_{ijk},

$$S_{IND} = \overline{P}_{CCF}\sum_{i=1}^{n}\sum_{j=1}^{\binom{n}{i}} C_{ij}\, p^i(1-p)^{n-i} \quad (17)$$

$$S_{CCF} \approx \sum_{i=1}^{n}\sum_{j=1}^{\binom{n}{i}} C_{ij}\frac{n}{\binom{n}{i}}\frac{\alpha_i^{(n)}}{\alpha_t} p = \frac{np}{\alpha_t}\sum_{i=1}^{n}\sum_{j=1}^{\binom{n}{i}}\frac{C_{ij}}{\binom{n}{i}}\alpha_i^{(n)} \quad (18)$$

$$S_{COMB} \approx \frac{np}{\alpha_t}\sum_{i=1}^{n}\sum_{j=1}^{\binom{n}{i}}\left[\frac{(1-C_{ij})}{\binom{n}{i}}\alpha_i^{(n)}\{\sum_{k=1}^{n-i} D_{ijk}\, p^k (1-p)^{n-i-k}\}\right] \quad (19)$$

4. Application to Popular System Configurations

Fig. 1 illustrates the unavailability and the SOP of 2-out-of-3, 2-out-of-4 and selective 2-out-of-4 systems. The unavailability and the SOP are identical in the case of 2-out-of-3 system. The selective 2-out-of-4 system shows

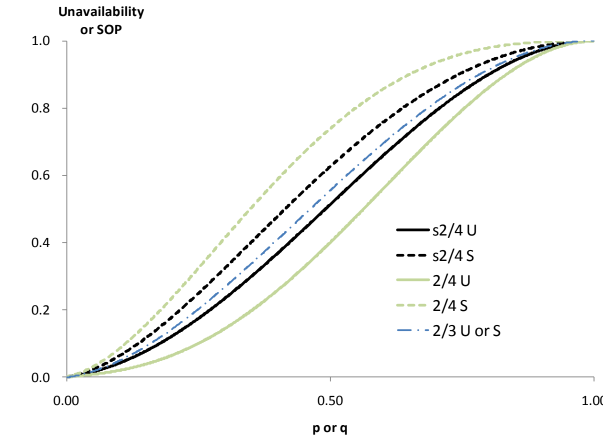

Fig. 1. The unavailability and the SOP of 2-out-of-3, 2-out-of-4 and selective 2-out-of-4 logics

the less unavailability but the greater SOP than 2-out-of-3 system. The 2-out-of-4 system shows the least unavailability but the greatest SOP. The independent failures dominate the system unavailability and SOP when *p* and *q* are larger values. However, since we consider the safety system failures, the region of small *p* and *q* is of interest. In this region, the CCF dominates the system unavailability and SOP. For example, at $p = q =$ 0.001 of selective 2-out-of-4 system, U_{IND} = 1.999e-6, U_{COMB} = 2.874e-7 and U_{CCF} = 1.869e-4 as in Fig.2.

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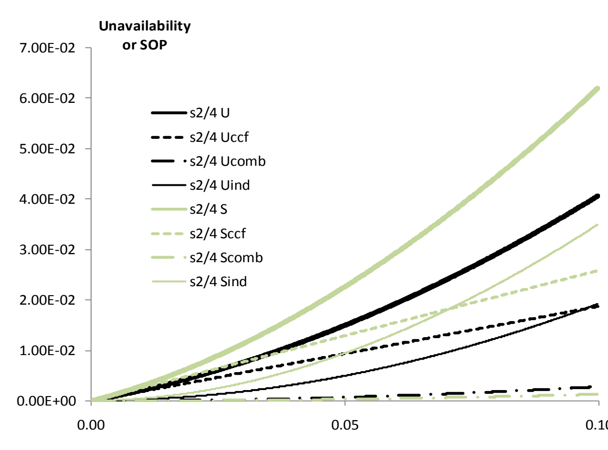

Fig. 2. The unavailability and the SOP of of selective 2-out-of-4 system when *p*,*q*<0.1