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An Investigation on the Monte-Carlo Simulation Method for the Hold-down Force and Margin Calculation

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1. Introduction

The hold-down forces and margins of the fuel assembly can be calculated deterministically or statistically. The deterministic method produces conservative results because the several design variables are combined by direct summation or SRSS(Square Root of the Sum of the Squares) method to calculate the hold-down forces and margins. The statistical method produces more realistic results by combining the input parameters statistically to calculate hold-down forces and margins.

The Monte-Carlo method is a widely used class of computational algorithms to simulate the behavior of various physical and mathematical systems. Because of the repetition of algorithms and the calculation of large number, the Monte-Carlo method is suited to computer simulation. This algorithm is often used to find solutions of mathematical problems (which may have many variables) that cannot easily be solved.

There are several methods for the generation of random numbers and statistical input variables that have a certain distribution. A large number of random numbers and input variables need to be generated for the Monte-Carlo simulation. In this study, the generation methods of random numbers and statistical input variables have been investigated. And, the minimum required number of random numbers and statistical input variables has been determined for the hold-down force and margin calculations.

2. Monte-Carlo Simulation

The Monte-Carlo simulation method provides approximate solutions to a variety of mathematical problems by performing statistical sampling experiments with a computer. The procedures for the Monte-Carlo simulation are as followings:

(1) Determination of statistical variables
(2) Generation of random numbers
(3) Generation of statistical variables
(4) Design analysis for physical and mathematical systems
(5) Statistical analysis for design analysis results

There are several methods that produce random numbers and input variables which have a certain distribution. The random number generation method for the current design program is that it divides probability distribution regularly and computes confidence intervals using the approximation of Abramowitz & Stegun[1]. And then, the random numbers are generated equally in each confidence interval.

The Inverse Transformation Method computes the reversed function of probability normal distribution function and produces random number. The Accept-Reject Method chooses the random number if it exists in the desired probability normal distribution. Otherwise, it is rejected. The Box-Muller Transformation Method produces two uniform random number and calculates the random deviates by given function[2].

3. Analysis

3.1 Determination of Statistical Variables

Several design variables and uncertainty factors are needed for the hold-down force and margin calculations. A series of sensitivity study were performed to select the statistical variables among design variables. The sensitivity studies were performed for the BOL(Beginning of Life) Hot and EOL(End of Life) Hot conditions and the Minitab Ver. 15 program[3] was used to perform correlation analysis.

3.2 Generation Method of Random Numbers

The Inverse Transformation Method is good to generate deviates for discrete distribution. But it is not applicable for probability distribution that is not easy to get inverse function. The Accept-Reject Method is easy to understand method and does not require inverse function. The Box-Muller Transformation Method has the lack of efficiency because it uses the sine and cosine function. The Accept-Reject Method was selected for the hold-down force and margin analysis based on the some sample test results.

3.3 Minimum Number of Random Numbers

The random numbers were generated using the Accept-Reject Method to determine the minimum required number of random numbers. The number of random numbers was selected from 1,000 to 200,000 with different increments. The hold-down margins were calculated 3 times for each selected random numbers using Accept-Reject Method. The results were compared with the calculation results using current design program.

4. Results and Discussion

4.1 Determination of Statistical Variables

Table 1 and 2 show the correlation analysis results for the statistical variables of the hold-down force and margin calculation, respectively. The variables in Table 1 and 2 were selected because it was assumed that these variables have an impact on the hold-down force and margin. Correlation coefficient represents the degree of correlation between two variables and P-value is a measure of how much evidence we have against the null hypothesis. Based on the sensitivity study and correlation analysis, the free length, caged length, spring stiffness, fuel assembly growth, and relaxation of the hold-down spring are determined as the statistical variables of the hold-down force calculation. The lift force and hold-down force are determined as the statistical variables of the hold-down margin calculation.

Table 1. Correlation Analysis Results for Hold-down Force Calculation

(a) at BOL Hot Condition

Variables	Correlation Coefficient(R)	P-Value
Free Length	0.609	0.000
Assembled Length	0.017	0.453
Caged Length	-0.422	0.000
Thermal Expansion	-0.137	0.000
Spring Deflection	0.735	0.000
Spring Stiffness	0.651	0.000

(b) at EOL Hot Condition

Variables	Correlation Coefficient(R)	P-Value
Spring Deflection(BOL)	0.429	0.000
Spring Deflection(EOL)	0.879	0.000
Assembly Growth	0.770	0.000
Spring Stiffness	0.460	0.000
Spring Relaxation	0.879	0.000

Table 2. Correlation Analysis Results for Hold-down Margin Calculation at EOL Hot Condition

Variables	Correlation Coefficient(R)	P-Value
Assembly Lift Force	-0.869	0.000
Assembly Weight	0.105	0.000
Required Force	-0.880	0.000
Hold-down Force	0.444	0.000

4.2 Generation Method of Random Numbers

Table 3 shows the random number generation results of the mean and standard deviation which follow the standard normal distribution N(0,1) using current design program and Accept-Reject Method. Fig. 1(a) and Fig. 1(b) show the histogram of 2,000 random numbers for current design program and 100,000 random numbers for Accept-Reject Method, respectively. The results show that the distribution of random numbers for current design program and Accept-Reject Method is close to the standard normal distribution N(0,1). The Accept-Reject Method has better accuracy than the current design program.

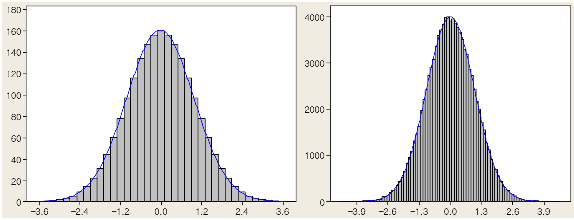
4.3 Minimum Number of Random Numbers

Fig. 2 shows the variation of the minimum hold-down margin as a function of the number of random numbers. The hold-down margins are highly unstable with the lower number of random numbers and still unstable with the numbers between 10,000 and 100,000. The hold-down margins start to be stable with the random number of higher than 100,000.

Table 3. Random Number Generation Results

Items		Values
Current Design Program*	Mean	0.0026
	Std. Dev.	0.9920
Accept-Reject Method *	Mean	0.0031
	Std. Dev.	0.9962
Accept-Reject Method**	Mean	0.00012
	Std. Dev.	1.00093

* with 2,000 random numbers, average of 10 results
** with 100,000 random numbers, average of 10 results



(a) Current Design Program (b) Accept-Reject Method
Fig. 1 Histogram of Random Numbers

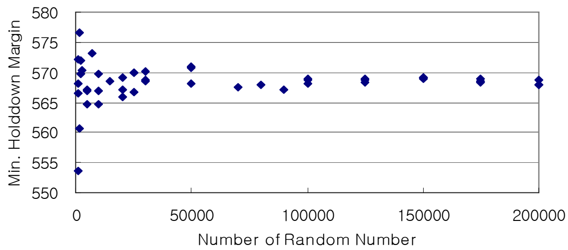


Fig. 2 Minimum Hold-down Margin Variation as a function of the number of random numbers

5. Conclusion

An investigation has been performed for the random numbers and statistical variables.

(1) The statistical variables were determined based on the sensitivity study and correlation analysis results.
(2) The Accept-Reject Method was selected for the generation of the random numbers and this method has better accuracy than the current design program.
(3) The minimum required number of the random numbers needs to be greater than 100,000 to have a stable result.

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