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LMM for Dynamic Response of a Structure Excited by Ground Acceleration

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1. Introduction

We considered two kinds of numerical modeling techniques for the dynamic response of a structure on the ground that is excited by a given acceleration time history. One of the techniques is the modeling based on the equation of motions relative to ground motion, and the other is a so-called large mass method(LMM). The former yields the relative response with respect to ground motion while the latter gives absolute motion of the structure including the ground motion.

The large mass method requires to allocate a large mass to the ground so that it causes the ground to move according to a given acceleration time history. In this paper, employing a spring-mass system, we analyzed the equations of motion of the two techniques and examined their differences.

2. Equation of Motion

Consider a single degree of freedom system shown in Fig. 1, where m represents the mass of a structure on the ground and $a(t)$ is a given acceleration of the ground.

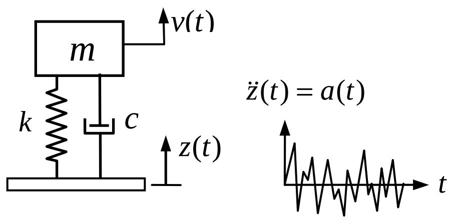


Fig. 1 Single-degree-of-freedom system

The equation of motion is given by

$$m(\ddot{w}+a)+c\dot{w}+kw=0 \tag{1}$$

where $w=v-z$.

Eq. (1) implies that the motion of mass m can be considered as the motion in the field of acceleration $a(t)$.

3. Large Mass Method

The same motion of mass m could be described by including the motion of the ground as shown in Fig. 2 if we could enforce the ground to move with the given acceleration by applying the force $F(t)=ma(t)$ to it.

The equation of motion of the structural mass and the ground is given by

$$\begin{bmatrix} M & 0 \\ 0 & m \end{bmatrix}\begin{Bmatrix} \ddot{z} \\ \ddot{v} \end{Bmatrix}+\begin{bmatrix} c & -c \\ -c & c \end{bmatrix}\begin{Bmatrix} \dot{z} \\ \dot{v} \end{Bmatrix}+\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}\begin{Bmatrix} z \\ v \end{Bmatrix}=\begin{Bmatrix} F(t) \\ 0 \end{Bmatrix} \tag{2}$$

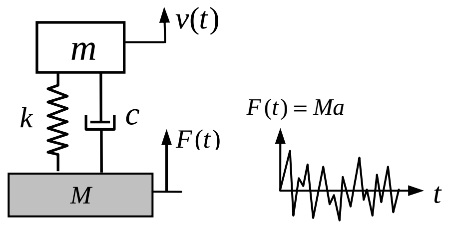


Fig. 2 Modeling of the S.D.O.F. system including the ground mass M

Summing the two equations in Eq. (2), we obtain the equation of motion of the entire system in Fig. 2,

$$\frac{\ddot{z}}{a}+\frac{\ddot{v}}{(M/m)a}=1 \tag{3}$$

The relationship between the accelerations, $\ddot{z}$ and $\ddot{v}$ is plotted in Fig. 3, from which the following useful theorems are drawn:

[Theorem A] As the value of M/m increases, the value of $\ddot{z}$ approaches to a and the magnitude of $\ddot{v}$ is not zero but a finite value.

[Theorem B] When $M/m\rightarrow\infty$, $\ddot{v}\equiv 0$ if $\ddot{z}\equiv a$.

[Theorem C] When $M/m\approx 1$, the magnitude of $\ddot{z}$ is less than a for $\ddot{z}\neq a$, or $\ddot{v}=0$ for $\ddot{z}=a$.

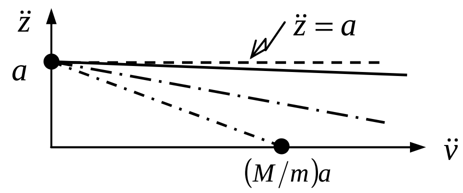


Fig. 3 Graph of the equation (3)

3. Numerical Examples

3.1 Models

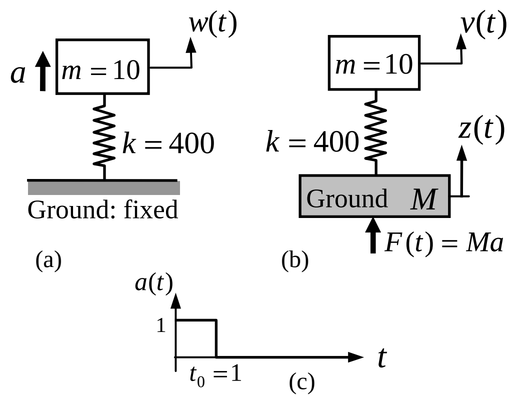


Fig. 4 (a) Model-1, (b) Model-2, and (c) ground acceleration

3.2 Numerical Examples

Table 1 Absolute accelerations($\ddot{z}$, $\ddot{v}$), velocities($\dot{z}$, $\dot{v}$) and displacements(z , v) of the model in Fig. 4(b), which is subjected to the acceleration in Fig. 4(c)

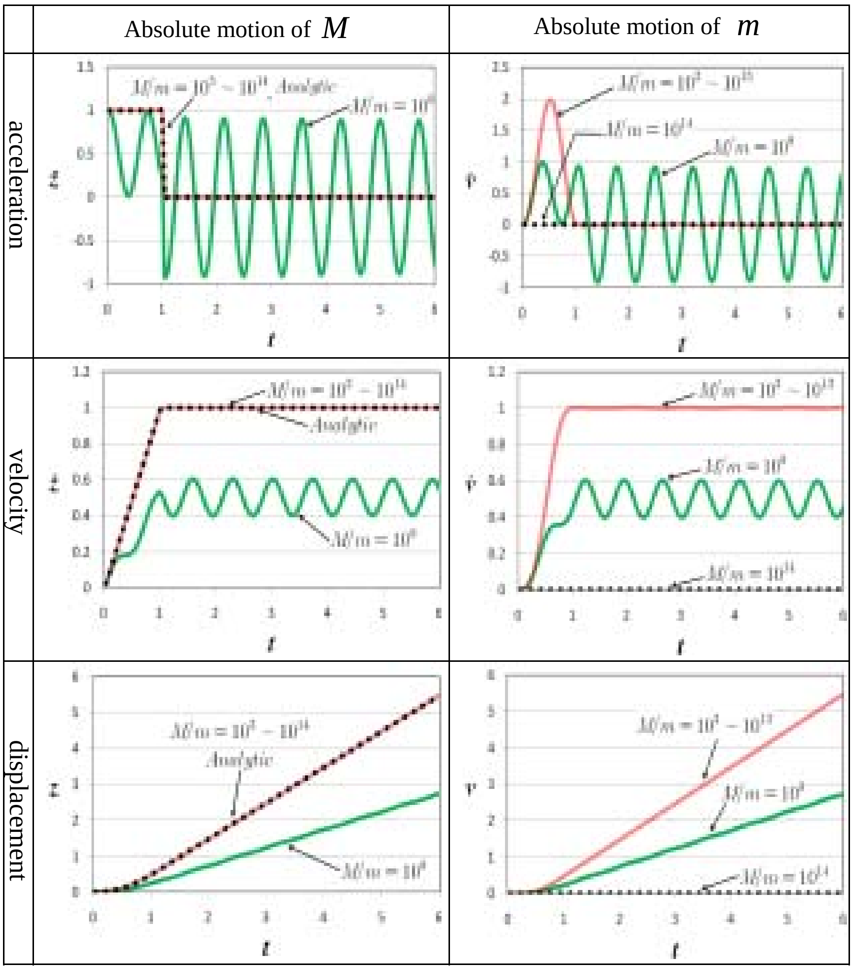
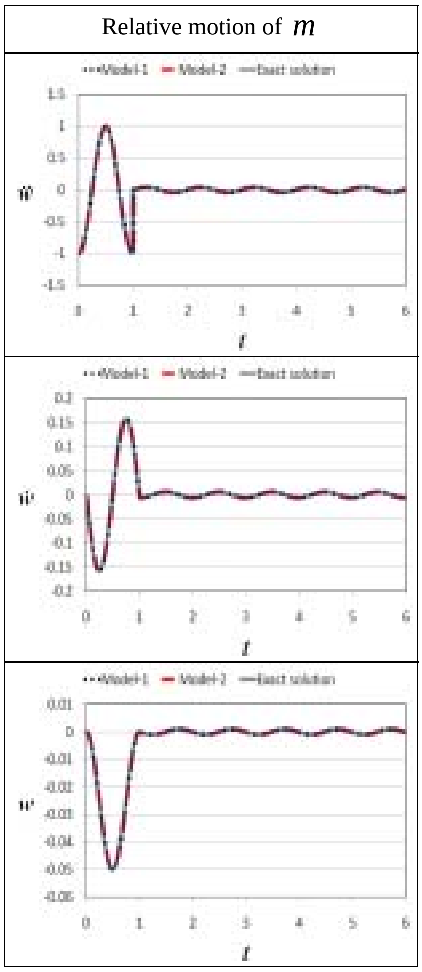


Table 2 Relative acceleration($\ddot{w}$), velocity($\dot{w}$) and displacement(w) of the models in Fig. 4 (a) and (b), which is subjected to the acceleration in Fig. 4(c) ($M/m = 10^5$)



3. Conclusions

We considered two kinds of numerical modeling techniques for the dynamic response of a structure on the ground, which is excited by an acceleration time history of the ground. One of the techniques is the modeling based on the equation of motions relative to ground motion, and the other is a so-called large mass method(LMM).

Employing a spring-mass system, we analyzed the equations of motion of the two techniques and examined their differences or equivalents through numerical tests.

(1) The large mass method requires that an appropriate large mass is input to produce accurate dynamic responses. The ground mass should be much larger than structural mass. But if the ground mass is too small or too large the LMM yields erroneous results.

(2) In LMM, an appropriate large ground mass causes the ground to move according to a given acceleration but a small value of the ground mass yield erroneous ground motion. And if the ground mass is too large the LMM gives zero dynamic response of the structure. This characteristic originates from its own intrinsic features.

(3) The two techniques give the same dynamic response in terms of the relative motion with respect to the ground. However, from the viewpoint of the mathematical analysis, it can be said that the LMM gives an approximate solution while the other model yields exact one.

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