Analytic Validation of the Heat Conduction Model of the Core Heat Transfer Model in the TASS/SMR

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1. Introduction

For the performance/safety analysis of the SMART (System-integrated Modular Advanced ReacTor) [1], under development in the Korea Atomic Energy Research Institute (KAERI), the TASS/SMR [2] (Transient And Setpoint Simulation/System-integrated Modular Reactor) code is also being developed and in the V/V (Verification/Validation) program. The validation of the TASS/SMR code can be divided into analytical and experimental ones.

The analytical validation can be accomplished for the models having analytic solutions. One of the representative models having analytic solutions in the TASS/SMR is the heat conduction model.

In this paper, the heat conduction model of the core heat transfer model in the TASS/SMR code is presented and validated with some analytic solutions for cylindrical geometry.

2. Heat Conduction Model of the Core Heat Transfer Model

The heat conduction equation for the core heat transfer model has the following simplified form, assuming only radial dependency of the temperature in space in the cylindrical coordinate system [3].

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial}{\partial r}\right)+q'''=\rho C\frac{\partial T}{\partial t} \tag{1}$$

where k and C denote the conductivity and the heat capacity of the fuel rod, respectively. And q''' is the volumetric heat generation rate in the fuel rod.

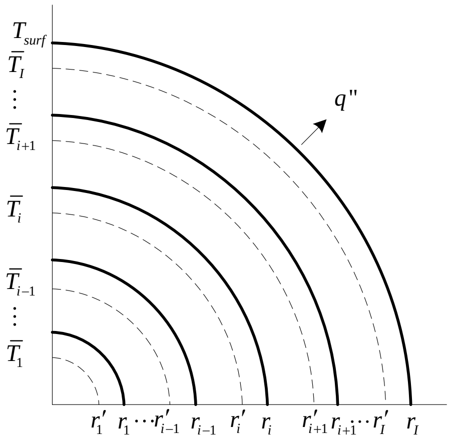

Fig. 1. Fuel rod mesh for the heat conduction equation in the core heat transfer model of the TASS/SMR code.

The conduction equation can be transformed into the difference equation by integration for the radial mesh, shown in the Fig. 1, assuming linearity of the fuel temperature in the mesh. The resulting equation is given as:

$$\begin{aligned}&-K_{i-1}A_{i-1}\Delta t\overline{T}_{i-1}^{n+1}+\left(M_iC_i+K_{i-1}A_{i-1}\Delta t+K_iA_i\Delta t\right)\overline{T}_i^{n+1}\\&-K_iA_i\Delta t\overline{T}_{i+1}^{n+1}=M_iC_i\overline{T}_i^{n}+Q_i\Delta t\end{aligned} \tag{2}$$

where K_i and A_i mean the conductivity per unit length and the surface area at the mesh interface i, respectively, while $\overline{T}_i^n$, M_iC_i and Q_i are the volume averaged temperature of the time step n, the heat capacity, and the heat generation rate of the mesh volume i, respectively. A fully implicit scheme is applied to calculate the average temperature of the n+1 time step.

3. Analytical Validation of the Heat Conduction Model of the Core Heat Transfer Model

The heat conduction equation of the core heat transfer model is validated by analytic solutions with cylindrical geometry and given boundary conditions.

Two cases are analyzed. One is the problem of having a time-independent boundary condition and a parabolic initial condition. The other is the problem of having a flat initial condition and a sinusoidal boundary condition.

The fuel rod, having a cylindrical shape, is assumed to be composed of a single material and has constant thermal properties and density. The radius of the fuel rod is set to be 1cm.

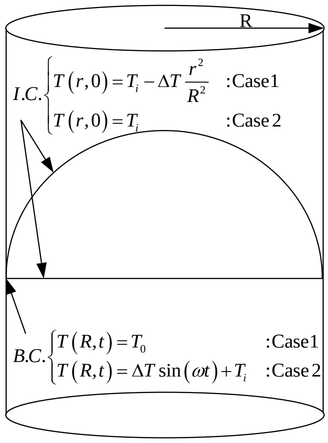

Fig. 2. Initial and boundary conditions for analytic validation of the heat conduction model of the core heat transfer model

3.1 Case 1

In case 1, the temperature has a parabolic distribution initially, and the outer surface is maintained at the constant temperature, as shown in Fig. 2.

The temperature difference ΔT in Fig. 2 means the difference between the rod center and the outer surface.

The analytic solution of case 1 is given as [4];

$$T(r,t)=\frac{4\Delta T}{R^2}\sum_{n=1}^{\infty}J_2(\lambda_n R)\frac{J_0(\lambda_n r)}{\lambda_n^2 J_1^2(\lambda_n R)}e^{-\alpha\lambda_n^2 t}+T_0 \quad (3)$$

where J_o and J_1 denote the Bessel function of the first kind, of order 0 and 1, respectively. And the λ_n's are the roots satisfying the following equation $J_o(\lambda_n R) = 0$.

α in the exponential term means the thermal diffusivity defined as $k/\rho C$.

In this case, the centerline temperature and outer temperature are assumed to be 500K and 300K, respectively. For the numerical analysis, the fuel rod is divided into 10 meshes and analyzed with given initial and boundary conditions. The comparison result is shown in Fig. 3. From the initial state, the simulation result shows agreement with the analytic solution and the temperature decreasing trends, as expected.

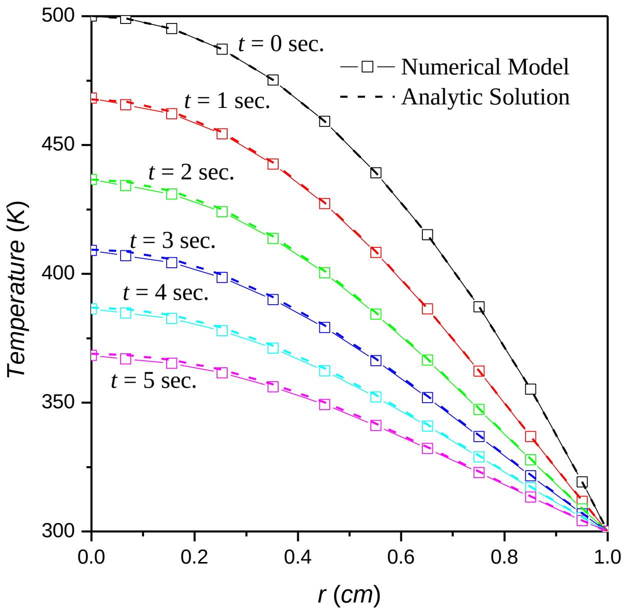

Fig. 3. Analytical Validation Results for the heat conduction model of the core heat transfer model : Case 1

3.2 Case 2

In case 2, the problem with the time-dependent boundary condition is treated. Initially, the temperature has a flat shape across the fuel rod. The sinusoidal boundary condition with frequency ω is imposed at the time zero, as shown in case 2 of Fig. 2.

The analytic solution for this case is given as [4];

$$T(r,t)=\sum_{n=1}^{\infty}\frac{\left[\alpha\lambda_n^2\left(e^{-\alpha\lambda_n^2 t}-\cos(\omega t)\right)-\omega\sin(\omega t)\right]}{\lambda_n\left(\alpha^2\lambda_n^4+\omega^2\right)RJ_1(\lambda_n R)\Big/2\Delta T\omega J_0(\lambda_n r)} \quad (4)$$

$$+T_i+\Delta T\sin(\omega t)$$

In this case, the initial temperature is set to 300K. The mesh number is the same as case 1. The comparison between the analytic solution and the numerical solution is shown in Fig. 4.

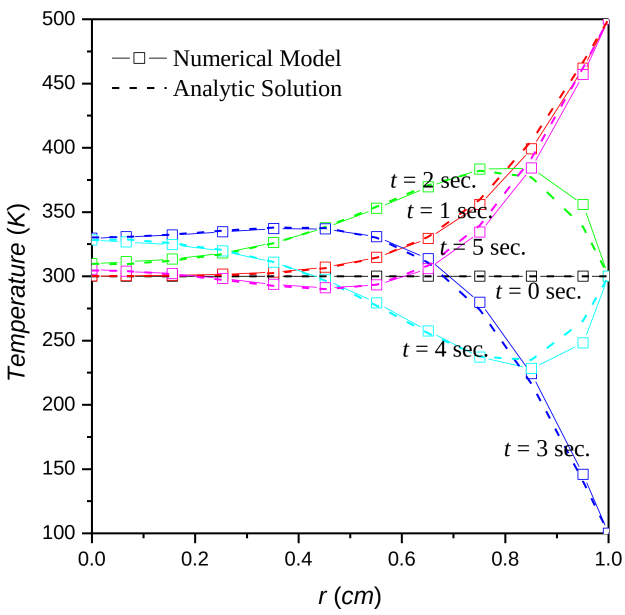

Fig. 4. Analytical validation results for the heat conduction model of the core heat transfer model : Case 2

As expected, the numerical results show agreement with the analytical solution for the time-dependent boundary condition.

3. Conclusions

Analytic validation of the heat conduction model of the core heat transfer model in the TASS/SMR has been accomplished for the known solutions in cylindrical geometry. Time dependent and independent boundary conditions are considered. The numerical results show agreement with the analytic solutions, as expected.

For further study, the analytic validations with heat source or experimental validations are recommended.

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