# Implementation of the First Collision Source Method in a Three-Dimensional, Unstructured Tetrahedral Mesh, Discrete-Ordinates Code

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## 1. Introduction

As with any discrete-ordinates code, ray-effects are an inherent problem, especially for shielding type problems with optically thin regions and localized (point) sources. Although ray-effects may be mitigated by increasing the quadrature order, this is often computationally prohibitive.

To mitigate ray-effects, many discrete-ordinates codes use first collision source methods. Such methods are characterized by a decomposition of the flux into its uncollided and collided components.

The uncollided flux is calculated analytically and the collided flux is calculated with the discrete-ordinates method.

In this paper, the implementation of the first collision source method in the MUST (Multi-group Unstructured geometry $S_N$ Transport) [1] code is presented and the results are compared with that of Attila [2,3].

## 2. Method and Results

### *2.1 Derivation of the Governing Equation*

The one-group transport equation [4] with vacuum boundaries and an isotropic point source at $\vec{r}_p$ is

$$\vec{\Omega}\bullet\nabla\psi(\vec{r},\vec{\Omega}) + \sigma_t(\vec{r})\psi(\vec{r},\vec{\Omega}) = \sum_{\ell=0}^{L}\sigma_{s,\ell}(\vec{r})\sum_{m=-\ell}^{\ell}Y^*_{\ell,m}(\vec{\Omega})\Phi^m_\ell(\vec{r}) + \frac{q_p}{4\pi}\delta(\vec{r}-\vec{r}_p), \quad (1)$$

where the moments of the angular flux is

$$\Phi^m_\ell(\vec{r}) = \int d\vec{\Omega}\, Y_{\ell,m}(\vec{\Omega})\,\psi(\vec{r},\vec{\Omega}). \quad (2)$$

Decompose the angular flux into two components as

$$\psi(\vec{r},\vec{\Omega}) = \psi^{(u)}(\vec{r},\vec{\Omega}) + \psi^{(c)}(\vec{r},\vec{\Omega}), \quad (3)$$

where $\psi^{(u)}$ is the uncollided angular flux and $\psi^{(c)}$ is the collided angular flux.

Rewrite Eq. (1) with Eq. (3) then, we have two equations

$$\vec{\Omega}\bullet\nabla\psi^{(u)}(\vec{r},\vec{\Omega}) + \sigma_t(\vec{r})\psi^{(u)}(\vec{r},\vec{\Omega}) = \frac{q_p}{4\pi}\delta(\vec{r}-\vec{r}_p), \quad (4a)$$

$$\vec{\Omega}\bullet\nabla\psi^{(c)}(\vec{r},\vec{\Omega}) + \sigma_t(\vec{r})\psi^{(c)}(\vec{r},\vec{\Omega}) = \sum_{\ell=0}^{L}\sigma_{s,\ell}(\vec{r})\sum_{m=-\ell}^{\ell}Y^*_{\ell,m}(\vec{\Omega})\Phi^{m,(c)}_\ell(\vec{r}) + q_s^{(u)}(\vec{r}). \quad (4b)$$

Eq. (4a) can be solved analytically for uncollided angular flux and $r_{\min}$ is introduced to prevent it from dividing by zero or very small distance.

$$\psi^{(u)}(\vec{r},\vec{\Omega}) = \begin{cases} \delta(\vec{\Omega}-\vec{\Omega}_r)\dfrac{q_p}{4\pi}\dfrac{e^{-\tau(\vec{r},\vec{r}_p)}}{\left|\vec{r}-\vec{r}_p\right|^2}, & \left|\vec{r}-\vec{r}_p\right| \ge r_{\min}, \\ \delta(\vec{\Omega}-\vec{\Omega}_r)\dfrac{q_p}{4\pi}\dfrac{1}{r_{\min}^2}, & \left|\vec{r}-\vec{r}_p\right| < r_{\min}. \end{cases} \quad (5)$$

The first collision source, $q_s^{(u)}$, is defined as

$$q_s^{(u)}(\vec{r}) = \sum_{\ell=0}^{L}\sigma_{s,\ell}(\vec{r})\sum_{m=-\ell}^{\ell}Y^*_{\ell,m}(\vec{\Omega})\Phi^{m,(u)}_\ell(\vec{r}). \quad (6)$$

The spherical harmonic moments of the uncollided angular flux, $\Phi^{m,(u)}_\ell$, is calculated as

$$\Phi^{m,(u)}_\ell = \int d\vec{\Omega}\, Y_{\ell,m}(\vec{\Omega})\,\psi^{(u)}(\vec{r},\vec{\Omega}) = \begin{cases} Y_{\ell,m}(\vec{\Omega}_r)\dfrac{q_p}{4\pi}\dfrac{e^{-\tau(\vec{r},\vec{r}_p)}}{\left|\vec{r}-\vec{r}_p\right|^2}, & \left|\vec{r}-\vec{r}_p\right| \ge r_{\min}, \\ Y_{\ell,m}(\vec{\Omega}_r)\dfrac{q_p}{4\pi}\dfrac{1}{r_{\min}^2}, & \left|\vec{r}-\vec{r}_p\right| < r_{\min}, \end{cases} \quad (7)$$

where $\tau(\vec{r},\vec{r}_p)$ is the optical distance between $\vec{r}$ and $\vec{r}_p$.

The optical distance from the point source to each node is calculated with

$$\tau(\vec{r},\vec{r}_p) = \sum_{i=1}^{N} r_i\sigma_{t,i}. \quad (8)$$

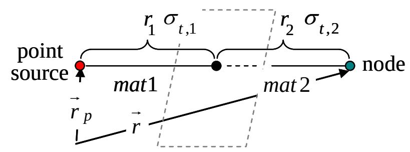


Fig. 1. The optical distance at the material interface.

### *2.2 Test and Results*

For verification, the test problem is configured as shown in Fig. 2 (point source is located at the corner of 10cm cube).

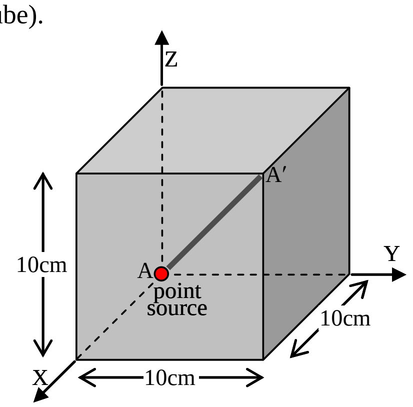


Fig. 2. The configuration of the test problem.

The unstructured tetrahedral mesh is generated by TetGen [5] code. The cross sections and parameters are listed in Table I. The reference scalar flux along the $AA'$ line is calculated by Attila with the identical conditions.

To compare the efficacy of the first collision source method, another run with very small volume source (identical source strength; 10#/sec) is performed with high $S_N$ order ($S_{16}$) and the result is shown in Fig. 3.

Table I: Cross sections and parameters

| | With first collision | Without first collision |
|---|---|---|
| # of cell | Attila: 39647<br>MUST: 39742 | MUST: 39837 |
| $\Sigma_{tot}$ ($cm^{-1}$)<br>$\Sigma_s$ ($cm^{-1}$) | 9.3e-2<br>1.0e-2 | |
| Point source (#/sec) | 10.0<br>at (0.05, 0.05, 0.05) | - |
| Volume source (#/$cm^3$-sec) | - | 10000.0<br>($0.1\times0.1\times0.1 cm^3$) |
| $S_N$ order | 4 | 16 |
| Error criterion | 1.0e-11 | |
| $r_{min}$ | 0.01 | - |

In Fig. 3, we can see 36 high-flux points on the opposite side of the source point location. These are the typical ray-effects phenomenon when the discrete ordinate calculation is performed with localized source in spite of using high $S_N$ order.

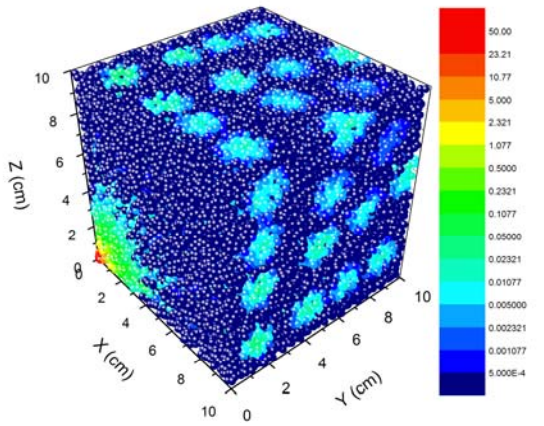


Fig. 3. Scalar flux without a first collision source method ($S_{16}$).

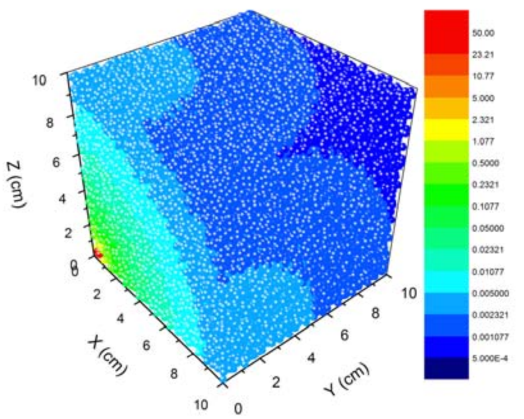


Fig. 4. Scalar flux with a first collision source method ($S_4$).

However, using the first collision source method, a very low $S_N$ order (even $S_4$) can resolve the solution, as can be seen from Fig. 4.

To check the calculation result, the scalar flux along $AA'$ line is compared with the result of Attila and shown in Fig. 5.

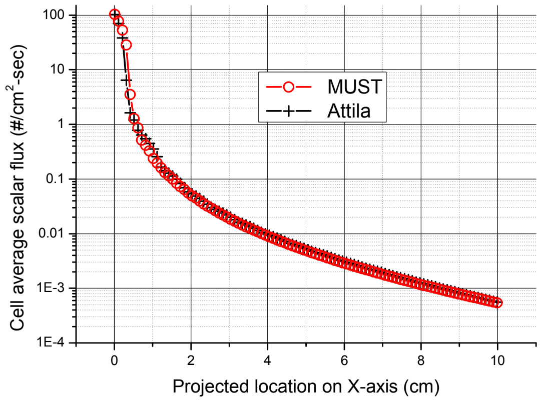


Fig. 5. The comparison of scalar fluxes along $AA'$ line.

The two scalar fluxes match well each other and we confirmed that the first collision source method is successfully implemented in the MUST code.

## 3. Conclusions

The first collision source method is widely used to mitigate ray-effects in the discrete ordinates code.

The typical ray-effects phenomenon is exampled by using the 3D simple test problem and the scalar flux distributions that are calculated with/without the first collision source method are compared.

The numerical test shows that the first collision source method which is implemented in the MUST code gives accurate solution.

## Acknowledgement

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