The Conventional and the Partial Current Based Unstructured Coarse Mesh Finite Difference Method on NEWT

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1. Introduction

The Coarse Mesh Finite Difference (CMFD) acceleration was devised to enhance the computational acceleration for the high order diffusion calculation such as nodal diffusion. And then it could be successfully applied to the acceleration of the transport eigenvalue calculations using Method of Characteristics (MOC). This method is quite effective for the fission source iteration by conserving the reaction rates inside each coarse mesh through the non-linear updating of the interface net currents from the high order transport equation. Fourier analysis for the fixed source and eigenvalue problems showed that the coupling of the high order transport and the low order CMFD calculations is not unconditionally stable.[1, 2] The partial current based CMFD (pCMFD) was devised to consider the interface partial currents for the better performance.[1] Fourier analysis for the eigenvalue problems showed that the coupling of the high order SC (Step Characteristics) transport and the low order pCMFD calculations is unconditionally stable.[2]

The NEWT[3] code is a multi-group discrete ordinate neutron transport code with flexible meshing capabilities. This code adopts the Extended Step Characteristic (ESC) approach for the arbitrary polygon meshes. In NEWT an acceleration scheme for the fission source iteration has been available only for the rectangular domain boundaries by using coarse mesh finite difference acceleration method only with rectangular coarse meshes.[4] Therefore no acceleration scheme could be applied to the wedge, triangle, hexagon and their symmetric domain boundaries. The conventional and the partial current based unstructured CMFD acceleration schemes (uCMFD and upCMFD) with the unstructured coarse meshes were implemented to be used for any domain boundaries.

2. Methods and Results

2.1 Unstructured CMFD and pCMFD

Figure 1 provides the geometrical configuration for the arbitrary polygons. In the uCMFD formulation, the interface net current is defined with the coupling coefficients ($\tilde{D}_i^k$) and the current corrective coefficients ($\hat{D}_i^k$) as follows:

$$J_i^k = -\tilde{D}_i^k(\phi_{i+k} - \phi_i) - \hat{D}_i^k(\phi_{i+k} + \phi_i), \tag{1}$$

and

$$\hat{D}_i^k = \frac{-\hat{J}_i^k - \tilde{D}_i^k(\phi_{i+k} - \phi_i)}{\phi_{i+k} + \phi_i}. \tag{2}$$

where $\hat{J}_i^k$ is the interface current from the high order transport calculation. With some algebra and a consideration of boundary conditions, the final conventional uCMFD equation is as follows:

$$\begin{aligned}\sum_{k=1}^{K}(\tilde{D}_i^k - \hat{D}_i^k)\Delta S_i^k\phi_i - \sum_{k=1}^{K}\varepsilon_k(\tilde{D}_i^k + \hat{D}_i^k)\Delta S_i^k\phi_{i+k}&, \\ + \Sigma_{t,i}\phi_i\Delta V_i = Q_i\Delta V_i&\end{aligned} \tag{3}$$

where

$$\varepsilon_k = 0 \quad (at\ boundary)\ and \quad \varepsilon_k = 1 \quad (at\ inside)\ .$$

In the upCMFD, the interface currents are defined with the positive and the negative current corrective coefficients ($\hat{D}_i^{k+}$ and $\hat{D}_i^{k-}$) instead of the net current corrective coefficients as follows:

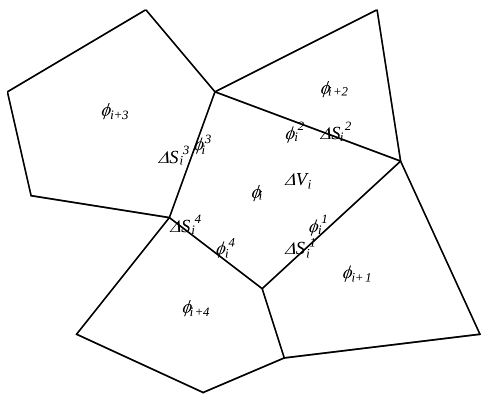

Figure 1. Unstructured coarse mesh configuration

$$J_i^k = -\tilde{D}_i^k(\phi_{i+k} - \phi_i) - (\hat{D}_i^{k-}\phi_{i+k} - \hat{D}_i^{k+}\phi_i)\,, \qquad (4)$$

$$\hat{D}_i^{k+} = \frac{2\hat{J}_i^{i+} + \tilde{D}_i^k(\phi_{i+k} - \phi_i)}{2\phi_i}\,, \qquad (5)$$

and

$$\hat{D}_i^{k-} = \frac{2\hat{J}_i^{k-} - \tilde{D}_i^k(\phi_{i+k} - \phi_i)}{2\phi_{i+k}}\,, \qquad (6)$$

where the coupling coefficients ($\tilde{D}_i^k$) are same as in the conventional uCMFD, and the partial currents ($\hat{J}_i^{k\pm}$) are from the high order transport calculations. With some algebra and a consideration of boundary conditions, the final upCMFD equation is as follows:

$$\sum_{k=1}^{K}\{\tilde{D}_i^k + \hat{D}_i^{k+} + \lambda_k(\hat{D}_i^{k+} - \hat{D}_i^{k-})\}\Delta S_i^k \phi_i - \sum_{k=1}^{K}\varepsilon_k(\tilde{D}_i^k + \hat{D}_i^{k-})\Delta S_i^k \phi_{i+k} + \Sigma_{t,i}\phi_i \Delta V_i = Q_i \Delta V_i\,, \qquad (7)$$

where

$$\lambda_k = 1 - \varepsilon_k$$
$$\varepsilon_k = 0\,(at\ boundary) \quad and \quad \varepsilon_k = 1\,(at\ inside)\,.$$

2.2 Geometrical constraints

In NEWT, coarse meshes for CMFD are constructed by overlapping the unstructured coarse mesh grids over the original constituent polygon meshes. In order to avoid generating tiny coarse meshes, some constraints are given at the coarse mesh construction as shown in Figure 2.

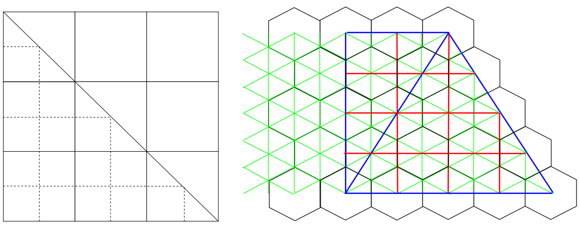

Figure 2. Coarse mesh constraints on NEWT

2.3 Calculation and results

Test calculations have been performed to see the effectiveness and the efficiency of the uCMFD and upCMFD. These problems include four fuel bundles with various geometries and compositions and water moderator. The 238-g AMPX master library, S_2 quadrature set for the high order ESC calculation and P_1 order of scattering matrix have been used. The computational results are shown in Table 1. While ESC+upCMFD is always rapidly convergent, ESC+uCMFD is very slow at case-3. If high order S_N quadrature sets are used, the computational efficiency will be increased much more.

3. Conclusion

Unstructured partial current based CMFD has been successfully implemented into NEWT. It is noted that the partial current based CMFD is more efficient and more stable than the conventional CMFD. And there is no additional computational burden in memory and computing time at all.

Table 1 Computational results

Case	Acceleration	Outer iteration	Time(sec)		$k_{eff.}$
			CMFD	Total	
1	No	50	-	120.669	1.0368070
	CMFD[a]	17	20.349	59.939	1.0368385
	uCMFD[b]	17	12.563	51.884	1.0368386
	upCMFD[c]	11	10.527	36.041	1.0368399
2	No	75	-	58.994	0.9526617
	uCMFD	22	9.856	26.541	0.9527445
	upCMFD	15	8.910	20.255	0.9527460
3	No	80	-	56.651	1.1046991
	uCMFD	189	142.128	280.289	1.1048481
	upCMFD	17	15.351	23.376	1.1047750
4	No	103	-	156.262	0.8285657
	uCMFD	17	21.981	49.754	0.8285751
	upCMFD	19	26.638	57.278	0.8285764

[a] CMFD acceleration with rectangular coarse mesh
[b] CMFD acceleration with unstructured coarse mesh
[c] Partial current based CMFD acceleration with unstructured coarse mesh

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