Examination of Triangle-based Polynomial Expansion Nodal Method in Fast Reactor Analysis

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1. Introduction

The triangle-based polynomial expansion nodal (TPEN) method involving a two-dimensional polynomial expansion within a triangle is an advanced nodal method to solve neutron diffusion problems in hexagonal-z geometry [1]. The TPEN method combines the higher order polynomial expansion nodal (HOPEN) [2] method and the nodal expansion method (NEM) by decoupling the three-dimensional neutron equation into a radial and an axial one. This method had already been implemented in the PARCS [3] code and its high accuracy was verified with various hexagonal benchmark problems for thermal reactors.

In this work, the TPEN method is employed in a multi-group spatial kinetics code FREK [4] for the fast reactor core analysis and its performance is examined for real fast reactor cores such as KALIMER-150/600. Since FREK can also be applied to thermal reactors, the VVER benchmark problems [5] are also analyzed.

2. TPEN Kernel and Cross Section Processor

The TPEN method solves two transverse-integrated neutron diffusion equations for the hexagonal-z geometry. One is the radial equation defined for a hexagon and the other is the axial equation defined for the z-direction which can be easily solved by NEM. The radial problem is solved by dividing the hexagon into six triangles and then by employing a polynomial expansion of flux within each triangle as follow:

$$\phi(x,y) = c_0 + a_x x + a_y y + b_x x^2 + b_u u^2 + b_p p^2 + c_x x^3 + c_u u^3 + c_p p^3 \quad (1)$$

where $u(x,y) = -\frac{1}{2}x - \frac{\sqrt{3}}{2}y$ and $p(x,y) = -\frac{1}{2}x + \frac{\sqrt{3}}{2}y$.

Nine unknowns per group in each triangle such as the nodal average flux, 3 corner fluxes, 3 surface average fluxes, one x- and one y-moment are defined. In the hexagonal point of view, there are 31 unknowns per group: 6 triangular node average fluxes, 6 *x*-moments, 6 *y*-moments, 6 inner surface fluxes, 6 outgoing partial currents and 1 center point flux. In order to determine these unknowns, 6 nodal balance equation, 6 x- and 6 y-weighted residual equations for each triangular node, 6 net current continuity conditions at inner surfaces, 6 incoming current condition at outer surfaces and one corner point leakage balance equation at the center point are used. The resulting linear system can be solved directly by a block Gaussian elimination scheme.

Since the one node TPEN calculation which updates the outgoing partial currents for use as the incoming currents of the neighboring nodes converges very slowly, a hexagonal CMFD formulation is employed to update the incoming partial currents more effectively using the current correction coefficient, $\hat{D}$, and the surface flux correction coefficient, β, which are determined in the previous nodal update step.

In fast reactor analysis, the ISOTXS cross section format is widely used. An ISOTXS cross section interpreter was incorporated in the FREK code so that the code can directly use existing fast reactor core models such as the DIF3D input decks. The macroscopic cross section generator of FREK called TRINX can treat the functional dependence of the microscopic cross sections on thermal condition parameters so that power distribution calculation with thermal feedback is possible with FREK coupled with the MARS system thermal-hydraulics code.

3. Performance Examination

In order to examine the performance of the TPEN module incorporated in FREK, two hexagonal-z problem sets have been analyzed. One is the VVER two-group benchmark problem and the other is KALIMER multi-group one.

2.1 VVER Benchmark Problems

The VVER-440/1000 3-D benchmark problems were analyzed. As shown in Table I, the maximum eigenvalue error is only 15 pcm and the maximum power distribution error is less than 1.5%, which confirms that the accuracy of the TPEN method is superior.

Table I. Two-Group Hexagonal Eigenvalue Problems

Problems	Eigenvalue Error [pcm]	Power Errors [%]	
		RMS	Maximum
VVER 440	3.9	0.26	1.00
VVER 1000	-15	0.91	1.55

2.2 KALIMER Core Analyses

In order to assess the practical calculation capability of FREK, KALIMER-150 and KALIMER-600 cores were analyzed. The basic information for the KALIMER core calculations, such as geometries, material compositions and microscopic cross sections and other group constants, were obtained from the

actual core model including the ISOTXS cross section files.

The KALIMER-150 core consists of 367 assemblies with a pitch of 16.186cm. The core height is 352.4 cm with the active core height of 105cm. The FREK model for the core was made with 30° symmetry, 32 planes with a thickness of about 10cm. For this core calculation, a 9 group cross section library was used. In contrast, the KALIMER-600 core consists of 703 assemblies with a pitch of 18.713cm. The core height is 359.36 cm with the active core height of 94c m. The FREK model for the core was made with 120° symmetry, 36 planes with a thickness of about 10cm. For this core calculation, a 25 group cross section library was used.

In order to obtain the reference solution for these cores, the DIF3D calculations performed with 54 and 96 triangles per hexagon and extrapolations using these two sets of results for asymptotic convergence in spatial method. Axial 1 and 2 axial meshes per node (~5cm) were used. The FREK calculation error is then compared with the DIF3D nodal calculation error.

As clearly shown in Table II, FREK has much higher accuracy than the DIF3D-nodal solution particularly for the eigenvalue. The results for the highly rodded cases indicates that the accuracy of TPEN is far superior than the corresponding DIF3D-nodal nodal method which involves error of ~500 pcm. Because of this large error, the FDM option is used in detailed design calculation of KALIMER at the expense of much longer computing time. With TPEN, a factor of 60 computing time saving is possible compared to the corresponding fine mesh FDM solution.

In table II, the relative assembly power distribution error of TPEN appears quite large for KALIMER-150. This is because there is a very low power region called IVS. The internal core power error is quite small as shown in Figure 1.

4. Conclusions

The TPEN method has been incorporated into FREK for the fast reactor neutronics analysis. Through VVER benchmark problems, the excellent solution accuracy of this method was confirmed in the power distribution as well as in eigenvalue. In KALIMER core analyses, the calculation capability of FREK for practical hexagonal core analysis was also verified. TPEN shows good accuracy even for the highly rodded cases so that a great time saving is possible with the nodal method by avoiding fine mesh FDM calculation which is currently inevitable in DIF3D because of the poor performance of its nodal method.

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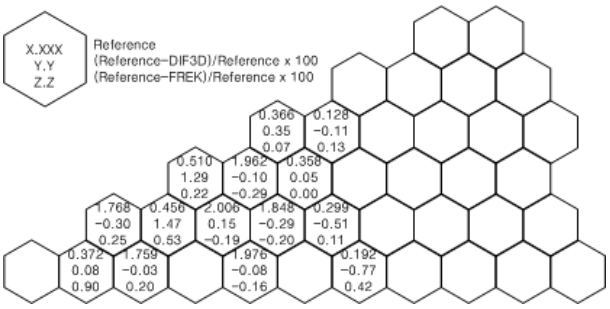

Fig 1. Relative Power Error of FREK for KALIMER-150

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Table II. Numerical Results of KALIMER Cores

		Reactivity Error [pcm]		Power Error [%], RMS(Maximum)				CPU Time (sec) [c]	
				All fuel regions [b]		Except IVS region			
		DIF3D	FREK	DIF3D	FREK	DIF3D	FREK	DIF3D -FDM [d]	FREK
KALIMER -150	Rod Out	-233	-52	7.49(25.56)	3.14(11.29)	0.78(1.31)	0.28(0.62)	548.8	1.7
	Rod In [a]	-457	-7	11.54(36.45)	5.89(20.96)	0.60(1.47)	0.34(0.90)	748.4	2.8
KLAIMER -600	Rod Out	-177	-30	2.70(3.23)	2.30(3.06)	-	-	3170.1	46.9
	Rod In	-576	84	3.09(5.54)	2.63(4.85)	-	-	3597.7	54.6

[a] All control assemblies including USS are injected fully.
[b] All fuel regions are considered in the power error calculation.
[c] CPU time on Intel Core2 Quad CPU Q8200, 2.33GHz
[d] 96 triangles per hexagon, axial 2 meshes per node (~5cm)