A Feasibility Study of CMFD Acceleration in Monte Carlo Eigenvalue Calculation

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1. Introduction

Monte Carlo methods are widely used because they are easy to implement and potentially very accurate. However, Monte Carlo methods have a drawback in high dominance ratio eigenvalue calculations (e.g., LWRs, NGNPs, etc.). When the dominance ratio becomes close to unity, the fission source distribution converges very slowly so that hundreds of cycles need to be skipped to obtain accurate results - regardless of how many particles are traced in each generation. A means of accelerating the source convergence of the distribution is strongly desired in such cases.

Recently, the coarse mesh finite difference (CMFD) which was originally known as the nonlinear acceleration method was successfully applied to accelerate the source convergence in MOC calculations[1,2]. This work is to examine the similar acceleration scheme to Monte Carlo calculations. Here, the CMFD linear system is obtained directly from MC tallies and the solution of the CMFD is used to adjust the source distribution of the MC calculation in the subsequent cycle. The performance of the proposed method is examined through a simple one-group high dominance ratio problem and also through a 2D multigroup problem. In an earlier work by Cho *et al.* [3], so called p-CMFD rebalance was applied externally for the acceleration of MCNP calculations as an initial trial of CMFD acceleration. A different CMFD formulation and strategy are proposed here with a simplified internal multigroup MC code.

2. Formulation of CMFD for MC acceleration

2.1. CMFD linear system from MC tallies

The key feature of CMFD linear system is to preserve the net current and node average flux of higher order system by introducing current correction coefficient $\hat{D}$ which is defined as:

$$\hat{D} = -\frac{J^{MC} + \tilde{D}(\bar{\phi}_R^{MC} - \bar{\phi}_L^{MC})}{(\bar{\phi}_R^{MC} + \bar{\phi}_L^{MC})} \quad (1)$$

where the superscript MC stands for the quantity obtained by MC tallies. After evaluating $\tilde{D}$, the net current in the CMFD system is formulated as follows:

$$J = -\tilde{D}(\bar{\phi}_R - \bar{\phi}_L) - \hat{D}(\bar{\phi}_R + \bar{\phi}_L) \quad (2)$$

This ensures that the node average flux and net current of CMFD system be the same as those of the MC solution if they are exact in the MC calculation.

However, the tallies of the MC calculation cannot be exact since they always have statistical errors. Even worse is that the tallies from not-stationary generation gives not-converged solutions. Even though the solutions from MC tallies have errors, CMFD solutions could be more accurate than MC itself as far as the global fission source distribution is concerned. This is because the CMFD system is very effective in solving elliptic problems. With CMFD, the perturbation occurred at a certain region can be propagated into the whole problem domain right away while MC simulation requires tenths of cycles for the propagation of the perturbation. Because of the statistical fluctuation of in the tallies of the node average flux and net current, it is necessary to accumulated tally information to obtain $\hat{D}$.

2.2. Weight adjustment of fission source

With the more accurate fission source distribution obtained by solving CMFD linear system, the MC fission source distribution can be adjusted which is possible by adjusting the weight of the fission source for the coarse mesh. The first step of weight adjustment is to obtain a PDF (Probability Density Function) from the CMFD fission source distribution as follows:

$$p_j = \frac{\int_{r_j} \psi(x)dx}{\sum_{j=1}^{J}\sum_{k=1}^{N_j}\int_{r_{k,j}} \psi(x)dx} \quad (3)$$

where *j* is the coarse mesh index while *k* is the fine mesh index in coarse mesh *j*. After obtaining the PDF, the expected value of fission source neutrons for each coarse mesh can be calculated as:

$$E[n_j] = Total\ \#\ of\ neutrons/cycle \times p_j \ . \quad (4)$$

With the updated number of source neutrons, generated in each mesh, the weight adjustment can be done as follows which can preserve the sum of fission source distributions:

$$\tilde{\omega}_{i,j} = \omega_{i,j} \times \frac{E[n_j]}{\sum_{i=1}^{n_j}\omega_{i,j}} \quad (5)$$

where $\tilde{\omega}_{i,j}$ and $\omega_{i,j}$ are the weight of *i*-th source neutron in coarse mesh *j* after and before the weight adjustment, respectively.

3. Performance Examinations of CMFD acceleration

The convergence of the MC fission source distribution can be estimated by Shannon entropy defined as:

$$H_{src} = -\sum_{j=1}^{N} P_j \cdot \ln_2(P_j) \quad (6)$$

where P_j is the fraction of the source distribution of Mesh j. In the following, the Shannon entropy behavior will be examined for high dominance ratio problems with the CMFD acceleration. The first test problem is the one that was examined by Larsen and Yang in their work of the functional MC method[4]. Among the several weakly coupled 1-D problems, a slightly modified problem identified by the following table is examined.

Table 1. Specification of Test Problem 1

Region	Location	Σ_t	Σ_s	$\nu\Sigma_f$
1	0<x<5	1.0	0.856	0.0
2	5<x<10	1.0	0.856	0.1968
3	10<x<15	1.0	0.856	0.0
4	15<x<20	1.0	0.856	0.19764
5	20<x<25	1.0	0.856	0.0

The following figure shows the Shannon entropy behavior of this test problem. It shows that at least 250 cycles are needed to be skipped to obtain correct MC solution. However, if the CMFD acceleration is applied, the fission source distribution converges much faster so that only 50 inactive cycles are required. Also a very stable behavior of the Shannon entropy is noted after the source convergence.

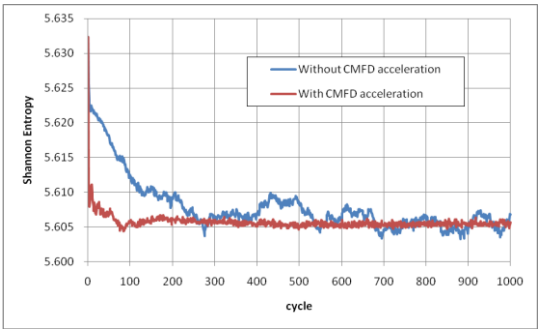


Fig 1. Shannon Entropy of the test problem 1 with/without CMFD acceleration (1million N/cycle)

Fig 2. shows the performance of the CMFD acceleration for Larsen's problem 3[4] which is a anisotropic scattering problem and its dominance ratio is about 0.999. For this the number of inactive cycles is reduced by about a factor of 10.

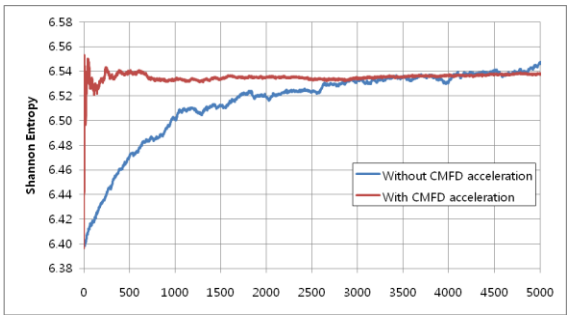
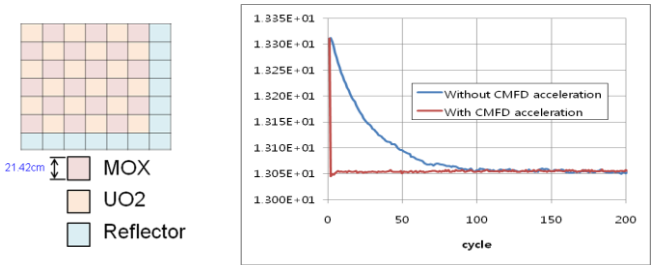


Fig 2. Shannon Entropy of the Larsen's problem 3 with/without CMFD acceleration (1million N/cycle)

This scheme is also effective in 2D eigenvalue problems. Test Problem 2 is a 2D 2G problem with homogenized assemblies. The quarter core configuration of the problem is shown in Fig 3. (a). The cross section data and geometrical configuration of this problem were obtained from the NEACRP L336 C5 configuration. The assembly pitch is 21.42 cm and the code has a dominance ratio of 0.956. Pin cell size meshes are chosen as the coarse mesh in this problem. As shown in Figure 3, the converged fission source distribution is obtained almost right away when the CMFD acceleration is applied.



(a) Configuration of test problem 2.

(b) Shannon Entropy VS. cycle of the test problem 2

Fig. 3 Configuration and acceleration results for test problem 2 (1million neutrons/cycle)

4. Conclusion

A CMFD acceleration scheme employing accumulated MC tallies for coarse mesh average fluxes and net currents was developed to achieve better convergence of the fission source distribution in MC eigenvalue problems. By the weight adjustment scheme, the global fission source distribution of the CMFD results is reflected into the MC source distribution. It was observed that the MC fission source distribution converges considerably faster with the CMFD acceleration. Further investigation of the CMFD acceleration for practical applications to 3-D continuous energy MC eigenvalue calculations is under way.

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