Cutset Quantification Error Evaluation for Shin-Kori 1&2 PSA model

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1. Introduction

Probabilistic safety assessments (PSA) for nuclear power plants (NPPs) are based on the minimal cut set (MCS) quantification method. In PSAs, the risk and importance measures are computed from a cutset equation mainly by using approximations. The conservatism of the approximations is also a source of quantification uncertainty. In this paper, exact MCS quantification methods which are based on the 'sum of disjoint products (SDP)' logic and Inclusion-exclusion formula are applied and the conservatism of the MCS quantification results in Shin-Kori 1&2 PSA is evaluated.

2. MCS Quantification Methods

2.1 General MCS Quantification Methods

In PSAs for NPPs, the risk measures (e.g., CDF and LERF) and importance measures are computed mainly by using "rare event" approximation or "min cut upper bound" approximation. If some measures are computed from a set of MCSs, $\{\mathbf{K}_i \mid i = 1, \ldots, m\}$ where $\mathbf{K}_i$ is the i-th MCS, by Rare event approximation (REA):

$$\Pr\{\bigcup_{i=1}^{m}\mathbf{K}_i\} \le \sum_{i=1}^{m}\Pr\{\mathbf{K}_i\}, \qquad (1)$$

and by Min cut upper bound approximation (MCUB):

$$\Pr\{\bigcup_{i=1}^{m}\mathbf{K}_i\} \le 1-\prod_{i=1}^{m}(1-\Pr\{\mathbf{K}_i\}). \qquad (2)$$

It is well known that these approximations always provide conservative results.

2.2 Inclusion-exclusion formula

The exact expression for MCS quantification is obtained by the so-called inclusion-exclusion formula:

$$\Pr\{\bigcup_{i=1}^{m}\mathbf{K}_i\} = \sum_{i=1}^{m}\Pr\{\mathbf{K}_i\} - \sum_{i=2}^{m}\sum_{j=1}^{i-1}\Pr\{\mathbf{K}_1\mathbf{K}_2\} + \cdots + (-1)^{m-1}\Pr\{\mathbf{K}_1\mathbf{K}_2\cdots\mathbf{K}_m\}. \qquad (3)$$

Eq. (3) can be bracketed by

$$\sum_{i=1}^{m}\Pr\{\mathbf{K}_i\} - \sum_{i=2}^{m}\sum_{j=1}^{i-1}\Pr\{\mathbf{K}_1\mathbf{K}_2\} \le \Pr\{\bigcup_{i=1}^{m}\mathbf{K}_i\} \le \sum_{i=1}^{m}\Pr\{\mathbf{K}_i\} - \sum_{i=2}^{m}\sum_{j=1}^{i-1}\Pr\{\mathbf{K}_1\mathbf{K}_2\} + \sum_{i=3}^{m}\sum_{j=2}^{i-1}\sum_{k=1}^{j-1}\Pr\{\mathbf{K}_1\mathbf{K}_2\mathbf{K}_3\}. \qquad (4)$$

The lower and upper bounds are approximate. For most problems it is relatively easy to obtain the lower and upper bounds.

2.3 Sum of Disjoint Products (SDP) methods

The SDP algorithms transform the set of cut sets into another set of mutually exclusive events (*DPs*) and then reduce the probability evaluation to a simple summation given as:

$$\Pr\{\bigcup_{i=1}^{m}\mathbf{K}_i\} = \Pr\{\mathbf{K}_1\} + \Pr\{\overline{\mathbf{K}_1}\mathbf{K}_2\} + \cdots + \Pr\{\overline{\mathbf{K}_1}\,\overline{\mathbf{K}_2}\cdots\mathbf{K}_m\} \qquad (5)$$
$$= \sum_{i=1}^{d}\Pr\{DP_i\}.$$

The developed SDP code [1] is based on the following recursive principle. Given a formula F, either F is reduced to a constant or it is possible to select a pivot variable x and to study recursively the two formulae $F_{\bar{x}}$ and F_x, i.e., the formula F in which the constants 0 and 1, respectively, are substituted for the variable x. In other words, the method builds, at least implicitly, a tree. Leaves of this tree encode constants. Internal nodes encode formulae of the form $F = xF_x + \bar{x}F_{\bar{x}}$. Branches of the tree that lead to a 1-leaf are labeled with wanted disjoint products. This tree-like presentation of the algorithm makes clear its exponential complexity.

As an illustration, let us consider the set of MCSs, $TOP = ab + bcd + dei + acei$, taken from Ref. 2. The binary tree traversed by the SDP code for TOP is shown in Figure 1. The sum of disjoint products developed by the SDP code is:

$$ab + a\bar{b}eid + a\bar{b}ei\bar{d}c + \bar{a}dbc + \bar{a}db\bar{c}ei + \bar{a}d\bar{b}ei.$$

All the produced DPs are mutually disjoint and their sum is equivalent to the original formula TOP.

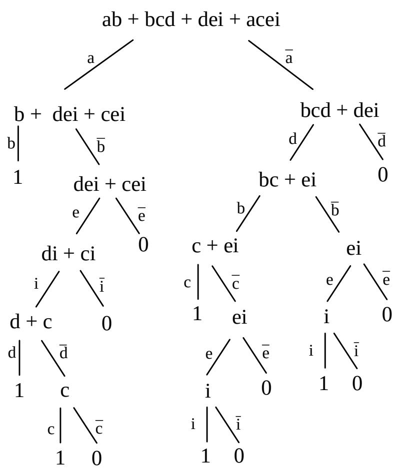

Figure 1. Binary tree built by the SDP algorithm

Unfortunately, for a large and complicated MCS problem, the SDP calculation is very time consuming.

3. Evaluation of MCS Quantification Errors

3.1 Proposed Method for usual PSA models

The MCSs of a PSA model can be classified into probabilistically disjoint groups of MCSs. For example, the MCS group of an initiating event is mutually exclusive with those of other initiating events. Therefore, the exact CDF can be written as:

$$CDF = \sum_{i=1}^{all} \left[F(IE_i) \times CCDP_i \right] \qquad (6)$$

where $F(IE_i)$ is the frequency of the *i*-th initiating event (IE_i) and $CCDP_i$ is the conditional core damage probability given that IE_i occurs. $CCDP_i$ can be exactly calculated by the SDP code and also approximated by equations (1), (2) and (4). Comparing with exact MCS quantification results, the quantification errors can be evaluated.

3.2 Internal Event PSA model for Shin-Kori Unit 1&2

The CDF model for Shin Kori 1&2 [3] has 16 initiating events. The cut-off value used in the PSA report is 10^{-11}. Table 1 shows the MCS quantification results based on REA.

Table 1: PSA quantification results

IE	*F*(*IE*)	CDF	# MCSs
I-SL	3.00E-3	1.65132E-6	1283
I-LOFW	5.50E-1	1.02485E-6	2189
I-LL	1.70E-4	7.91489E-7	1047
I-GTRN	3.40	5.37823E-7	1607
I-SGTR	4.50E-3	5.87704E-7	912
I-ML	1.70E-4	5.07377E-7	801
I-LOOP	1.59E-2	4.09804E-7	2489
I- RVR	2.66E-7	2.66E-7	1
I- LOCCW	7.30E-2	2.48014E-7	1550
I-LODC	3.50E-3	1.99896E-7	1082
I- SBO	1.59E-6	3.41262E-8	28
I- LOCV	2.30E-1	6.97051E-8	204
I-LSSB	1.50E-3	1.05996E-8	124
I-LOKV	1.75E-3	2.52027E-9	73
ISL	1.77E-9	1.77E-9	1
I-ATWS	2.17E-5	2.78664E-7	59

Using the MCSs of each IE, its corresponding CCDP can be calculated by the equations (1), (2), (4), and (5). Using the equation (6), the total CDF can be also calculated. The SDP method provides exact MCS quantification. Comparing one MCS quantification result to another, its MCS quantification error can be evaluated.

Table 2 provides MCS quantification results of the methods (REA, MCUB, SDP, lower bound and upper bound of the so-called *inclusion-exclusion formula*) expressed in percentage terms. Some SDP results (for I-SL and I-LOFW) in Table 2 are not obtained due to long CPU time. Table 2 shows that the SDP results are exact and the upper bounds of the equation (4) are reasonably good. Based on the upper bounds of the equation (4), it is sure that the total CDF for internal events of Shin-Kori unit 1&2 calculated by REA is overestimated by more than *1.737 %* of the exact one. In other words, the exact total CDF based on the identified MCSs is less than *98.263 %* of the reported value obtained by REA.

Table 2: Comparison of MCS quantification with REA

IE	**MCUB** By Eq.(2)	**SDP** By Eq.(5)	**Lower** of Eq.(4)	**Upper** of Eq.(4)
I-SL	99.981 %	? %	97.976 %	98.036 %
I-LOFW	100.00 %	? %	99.184 %	99.208 %
I-LL	99.784 %	99.073 %	99.049 %	99.092 %
I-GTRN	100.00 %	99.126 %	99.095 %	99.132 %
I-SGTR	99.994 %	? %	98.425 %	98.460 %
I-ML	99.870 %	99.304 %	99.282 %	99.307 %
I-LOOP	99.999 %	? %	95.566 %	95.899 %
I-RVR	100 %	100 %	100 %	100 %
I-LOCCW	100.00 %	93.450 %	92.985 %	93.615 %
I-LODC	99.997 %	? %	91.567 %	91.997 %
I-SBO	99.553 %	99.531 %	99.529 %	99.531 %
I-LOCV	100.00 %	99.500 %	99.494 %	99.501 %
I-LSSB	100.00 %	99.331 %	99.325 %	99.332 %
I-LOKV	100.00 %	95.289 %	95.146 %	95.331 %
ISL	100 %	100 %	100 %	100 %
I-ATWS	99.777 %	99.741 %	99.740 %	99.741 %
Total CDF	99.945 %	? %	98.274 %	98.263 %

4. Conclusions

This paper proposed an approach to evaluate the MCS quantification errors and showed the application into a real NPP PSA model. From this study, it is known that for large MCS problems (e.g., NPP PSA problems), the 3rd order approximation of the inclusion-exclusion formula is applicable and reasonably exact.

REFERENCES

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