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On the Form of Momentum Equations for Multi-Dimensional Two-Phase Flow

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1. Introduction

Some of thermal hydraulics codes for multi-dimensional two-phase flow analysis use the non-conservative form of the momentum equations. For example, the three-dimensional thermal hydraulic modules of the system codes, such as RELA5-3D [1], MARS [2], TRAC-PF1 [3], and TRACE [4], use the non-conservative form. Fine-scale two-phase flow codes, such as the NEPTUNE CFD [5] and ACE-3D codes [6], also adopt the non-conservative form. Meanwhile, the computational fluid dynamics codes, such as FLUENT [7], CFX [8], and STAR-CD [9], use the conservative form of the momentum equations.

From a mathematical point of view, the momentum equations in the non-conservative form are equal to those in the conservative form. However, they are different in numerical integrations. This difference may evoke inaccurate numerical solutions under some two-phase flow conditions. In this paper, we suggest the use of momentum equations in a semi-conservative form [10] instead of the non-conservative form, which is close to the conservative form but still maintains the advantage of the non-conservative form

2. Various Forms of the Momentum Equations

The above-mentioned codes use the two-fluid model for two-phase flows. The momentum equation of k-phase in a conservative form is

$$\frac{\partial}{\partial t}(\alpha_k\rho_k\vec{u}_k)+\nabla\cdot(\alpha_k\rho_k\vec{u}_k\vec{u}_k)=\vec{F}_k, \tag{1}$$

where $\vec{F}_k$ includes the pressure gradient, viscous and turbulent shear forces, body force, and interfacial forces. The continuity equation of k-phase is

$$\frac{\partial}{\partial t}(\alpha_k\rho_k)+\nabla\cdot(\alpha_k\rho_k\vec{u}_k)=\vec{\Omega}_k, \tag{2}$$

where $\vec{\Omega}_k$ is the interfacial mass transfer rate per volume. In the above-mentioned codes, the momentum equations in a conservative form are expanded so that they are solved in a non-conservative form and the velocity of each phase is used as a primary unknown. Expanding the left-hand side (LHS) of Eq. (1) and substituting Eq. (2) into Eq. (1) yields the momentum equations in the non-conservative form:

$$\alpha_k\rho_k\frac{\partial\vec{u}_k}{\partial t}+\alpha_k\rho_k\vec{u}_k\cdot\nabla\vec{u}_k+\vec{u}_k\Omega_k=\vec{F}_k \tag{3}$$

Equation (3) can be used to advance the velocity components directly, instead of $\alpha_k\rho_k\vec{u}_k$. Integrating the convection term of Eq. (1) over a control volume is clearly defined. However, the integration of the second term in the LHS of Eq. (3) requires some assumptions, such as the use of the cell-centered approach for $\alpha_k\rho_k\vec{u}_k$. This difference may evoke inaccurate numerical solutions under some two-phase flow conditions.

Jeong et al. [10] investigated this problem and suggested a new discretization method, which is based on the conservative form of the momentum equations. This yielded improved numerical results for heterogeneous two-phase flows. Recently Park et al. [11] suggested the use of a semi-conservative form of the momentum equations to resolve this problem. In this approach, only the temporal term of Eq. (1) is expanded and the continuity equation, Eq. (2), is substituted. Then we obtain the semi-conservative form:

$$\alpha_k\rho_k\frac{\partial\vec{u}_k}{\partial t}+\nabla\cdot(\alpha_k\rho_k\vec{u}_k\vec{u}_k)-\vec{u}_k\nabla\cdot(\alpha_k\rho_k\vec{u}_k)+\vec{u}_k\Omega_k=\vec{F}_k \tag{4}$$

Numerically integrating Eq. (4), the second term in the LHS entails no error and the third term yields small error. However, this integration error is generally smaller than that for integrating the second term in the LHS of Eq. (3). It is noted that, for a steady-state flow, Eq. (4) becomes Eq. (1), i.e., the semi-conservative form becomes the conservative form.

Comparing the two methods mentioned above, they are basically same. For example, in the case of positive-direction flow in x- and y-directions (see Fig. 1), the new discretization method, based on the conservative form [10], gives:

$$\left(\overline{\alpha\rho}\right)^n_{I,j}\frac{U^{n+1}_{I,j}-U^n_{I,j}}{\delta t}+(\alpha\rho)^n_{i,j}U^{n+1}_{I,j}\frac{\left(U^{n+1}_{I,j}-U^n_{I-1,j}\right)}{\delta x}+\left(\overline{\alpha\rho}\right)^n_{I,j-1}\overline{V}^n_{I,J-1}\frac{\left(U^{n+1}_{I,j}-U^n_{I,j-1}\right)}{\delta y}$$
$$=\left(F_x-U\Gamma\right)^{n+1}_{I,j} \tag{5}$$

Meanwhile, by using the semi-conservative form, we can obtain the following discretized equation [11]:

$$(\overline{\alpha\rho})^n_{I,j}\frac{U^{n+1}_{I,j}-U^n_{I,j}}{\delta t}+(\alpha\rho)^{n+1}_{i,j}U^{n+1}_{I-1,j}\frac{U^{n+1}_{I,j}-U^{n+1}_{I-1,j}}{\delta x}+\left(\overline{\alpha\rho}\right)^{n+1}_{I,j-1}V^{n+1}_{I,J-1}\frac{U^{n+1}_{I,j}-U^{n+1}_{I,j-1}}{\delta y}$$
$$=(F_x-U\Gamma)^{n+1}_{I,j}\cdot \tag{6}$$

Only the coefficients of the second term in Eq. (5) and (6) are different. This is due to neglecting the second-order terms in the course of deriving Eq. (5). Otherwise, Eq. (5) and Eq. (6) are exactly the same.

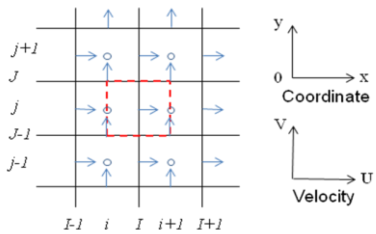


Fig. 1. The computational grid.

3. Comparison of the Numerical Results

The effects of the form of momentum equations were assessed against various conceptual problems using the CUPID code [11]. For single-phase flows, both the non-conservative and conservative form yield almost the same results. However, for two-phase flow, the results were significantly different especially for heterogeneous two-phase flows. Two examples are presented below.

3.1 Air-water phase separation

A conceptual phase separation problem was calculated. A two-dimensional vertical plane with 1 m in height and 1 m in width is considered. Uniform 80 x 80 meshes were used for this problem. Initially, the vertical plane is filled with a homogeneous two-phase mixture with a void fraction of 0.5. At t=0 s, the fluid is set in motion by gravity: the gas phase goes up and the liquid moves down. At t=4.8 s, the phase separation is completed. Figure 2 compares void distributions at x=0.5 m. The solutions with the semi-conservative form are closer to the analytical solutions.

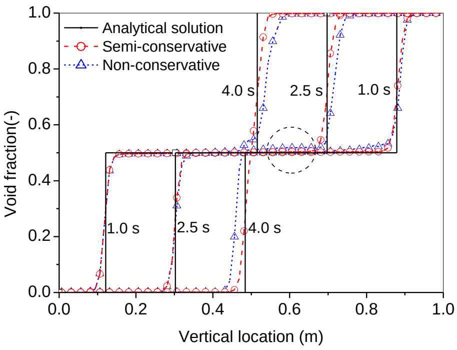
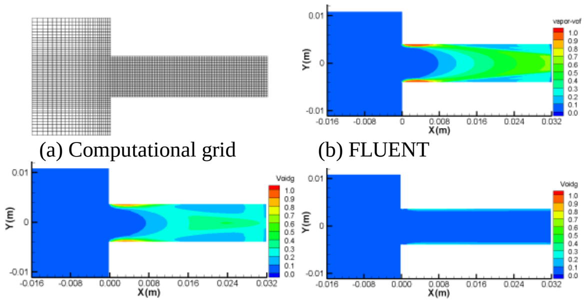


Fig. 2. Void distributions of the phase separation problem.

3.2 Cavitations with a sudden contraction

Cavitations with a sudden contraction was simulated. Figure 3 (a) shows the schematic of the computational grid. The length and height of the left and right parts are (0.016 m x 0.02304 m) and (0.032 m x 0.008 m), respectively. The entire domain was modeled with 3875 cells. The left and the right ends are the pressure boundaries with 0.5 MPa and 0.095 MPa, respectively. Other boundaries are walls with no-slip conditions. The water temperature is 346.1 K. These flow conditions were chosen so that cavitations occur near the throat.

This problem was also simulated by using the FLUENT code which has a two-fluid model with the fully conservative form of momentum equations. Figures 3 (b) through (d) show the steady-state void distributions of the three calculations. Cavitations occur near the throat after flow separation at the sharp edge. The result of the semi-conservative form is similar to that of FLUENT: the void fraction near the walls increases to the maximum at x = ~0.0034 m. Maximum local void fractions of the semi-conservative CUPID and FLUENT codes are 0.91 and 1.0, respectively. Figure 3 (d) shows the void distribution with the non-conservative form is quite different from the others.



(a) Computational grid (b) FLUENT
(c) CUPID: Semi-conservative (d) CUPID: Non-conservative
Fig. 3. Computational grid and steady-state void distributions.

4. Conclusions

The numerical effects of the form of the momentum equations have been investigated, and the use of momentum equations in a semi-conservative form instead of the non-conservative form was suggested. To study the numerical effects, various conceptual problems were simulated and the advantages of the semi-conservative form against the non-conservative form were clearly shown. The semi-conservative form can be easily implemented in any two-phase code that uses the non-conservative form of the momentum equation. Furthermore, this can be easily adapted into structured or non-structured grids. Therefore, the semi-conservative form of the momentum equations is recommended for the two-phase flow codes that adopt the non-conservative form.

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