Temperature Calculation of Annular Fuel Pellet by Finite Difference Method

Yong Sik Yang, Je Geon Bang, Dae Ho Kim, Sun Ki Kim, Ik Sung Lim, Kun Woo Song
Korea Atomic Energy Research Institute, 1045, Daedeok-daero, Yuseong-gu, Daejeon, 305-303, Korea
yys@kaeri.re.kr

1. Introduction

KAERI has started an innovative fuel development project for applying dual-cooled annular fuel to existing PWR reactor[1]. In fuel design, fuel temperature is the most important factor which can affect nuclear fuel integrity and safety.

Many models and methodologies, which can calculate temperature distribution in a fuel pellet have been proposed. However, due to the geometrical characteristics and cooling condition differences between existing solid type fuel and dual-cooled annular fuel, current fuel temperature calculation models can not be applied directly[2].

Therefore, the new heat conduction model of fuel pellet was established. In general, fuel pellet temperature is calculated by FDM(Finite Difference Method) or FEM(Finite Element Method), because, temperature dependency of fuel thermal conductivity and spatial dependency heat generation in the pellet due to the self-shielding should be considered.

In our study, FDM is adopted due to high exactness and short calculation time.

In our modeling, there are some assumptions to solve the heat conduction equation as follows:

- Heat conduction in the azimuthal direction is ignored (axisymmetric)
- Heat conduction in the axial direction is considered negligible relative to radial heat conduction
- Steady-state heat conduction

Based on the above assumptions, 1-D steady-state heat conduction equation in cylindrical coordinate system is developed and verified.

In this paper, detailed modeling by FDM is introduced and its verification result is summarized.

2. Modeling of Pellet Heat Conduction Equation

For the application of FDM to heat conduction equation of fuel pellet, a fuel pellet is subdivided into concentric annular elements of equal distance. Node 1 is located in the pellet's inner surface and node M is located on the outer surface.

Fig. 1 represents three typical mesh points in the pellet. Where, r and l means quantities to the right and left respectively. The k is thermal conductivity, δ is mesh point distance and S indicates a heat source. If we select m-th node as a standard, heat conduction calculation is applied to the volume and surfaces defined by dashed line in Fig. 1.

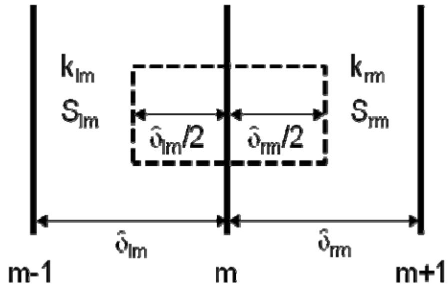

Fig. 1 Typical mesh points

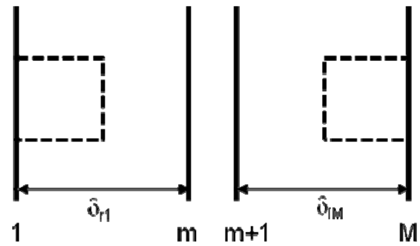

Fig. 2 Boundary mesh point

Fig. 2 shows boundary mesh points. At node 1, left side quantities are zero and right side quantities go to zero at node M.

1-D steady state heat conduction equation in cylindrical coordinate system is described as follows[3].

$$\iiint_v \nabla k(T,r) \bullet T(r) dV + \iiint_v S(r) dV = 0 \qquad (1)$$

By finite difference numerical approximation, the first term of can be approximated by

$$\iiint_v \nabla k(T,r) \bullet T(r) dV = \iint_s k(T,r) \nabla T(r) \bullet d\bar{S} \approx$$
$$-\left[(T_{m-1} - T_m) k_{lm} \delta^s_{lm} + (T_{m+1} - T_m) k_{rm} \delta^s_{rm}\right] \qquad (2)$$

and the second term is approximated by

$$\iiint_v S(r) dV \approx S_{lm} \delta^v_{lm} + S_{rm} \delta^v_{rm} \qquad (3)$$

where, following terms are defined.

$$\delta^v_{lm} = \pi \delta_{lm} \left(r_m - \frac{\delta_{lm}}{4} \right), \ \delta^v_{rm} = \pi \delta_{rm} \left(r_m + \frac{\delta_{rm}}{4} \right)$$

$$\delta^s_{lm} = \frac{2\pi}{\delta_{lm}} \left(r_m - \frac{\delta_{lm}}{2} \right), \ \delta^s_{rm} = \frac{2\pi}{\delta_{rm}} \left(r_m + \frac{\delta_{rm}}{2} \right)$$

where, v and s means volume and surface-gradient weights, which must be considered to calculate quantities such a thermal conductivity and heat generation.

Gathering Eq. (2) and (3), the basic difference heat conduction equation for the m-th mesh point is

$$(T_{m-1}-T_m)k_{lm}\delta^s_{lm}+(T_{m+1}-T_m)k_{rm}\delta^s_{rm}=S_{lm}\delta^v_{lm}+S_{rm}\delta^v_{rm} \quad (4)$$

Writing Eq. (4) in condensed form, the finite difference approximation for the m-th mesh point is

$$a_mT_{m-1}+b_mT_m+C_mT_{m+1}=d_m \quad (5)$$

where, $a_m=-(k_{lm}\delta^s_{lm})$, $b_m=-a_m-c_m$, $c_m=-(k_{rm}\delta^s_{rm})$ and $d_m=S_{lm}\delta^v_{lm}+S_{rm}\delta^v_{rm}$.

To solve the Eq. (5), we need boundary condition. In this work, inner surface(T_i) and outer surface(T_o) temperatures are known. Detailed procedure to derive pellet surface temperature is summarized in reference [4].

Under boundary condition, Eq. (5) has tridiagonal matrix shape and can be solved easily by Thomas algorithm.

3. Model Verification

To confirm an appropriateness of the established model, verification study was performed. By using a same input condition, calculated temperature profile by FDM is compared with analytical solution of heat conduction equation(Eq. (6)). However, analytical solution can not consider temperature dependency of thermal conductivity and spatial dependent heat generation. Therefore, thermal conductivity was fixed as constant and radially uniform heat generation condition was used to calculate temperature profile by a new model.

Analytical solution of 1-D steady state heat conduction equation at cylindrical coordinates can be expressed as Eq. (6)[4].

$$T(r)=T_i-\frac{q'''(r^2-r_i^2)}{4k_f}+\left[\frac{(T_o-T_i)+\frac{q'''}{4k_f}(r_o^2-r_i^2)}{\ln\frac{r_o}{r_i}}\right]\ln\left(\frac{r}{r_i}\right) \quad (6)$$

where, q''' is volumetric heat generation rate, r_i is pellet inner surface, r_o is pellet outer radius and k_f is fuel thermal conductivity.

The detailed input condition is summarized in Table. I.

Fig. 1 shows the verification result of the new model. Calculated radial temperature profile shows good agreement with the analytically calculated one.

The maximum relative error of calculated temperature between new model and analytical solution is within ±0.1%

Table I: Input for FDM model verification

Input variable		Value
Linear power rate	kW/m	60
Pellet surface Temp. [inner/outer]	℃	714.6/678.5
Pellet outer radius	mm	7.02
Pellet inner radius	mm	4.9
Fuel thermal conductivity	W/m-K	4

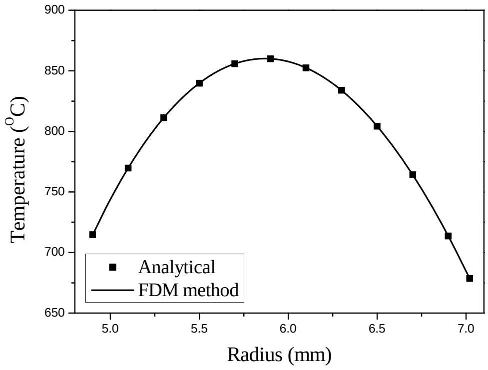

Fig. 1. Radial temperature profile

3. Conclusion

A new model, which can calculate pellet temperature of dual-cooled annular fuel, was developed. The new model is derived from 1-D steady state heat conduction equation in cylindrical coordinate system.

To consider a spatial distribution of heat generation in pellet and temperature dependency of thermal conductivity, implicit finite difference method was used.

Under simplified conditions such a radially uniform heat generation and constant pellet thermal conductivity, the new model was verified and shows good agreement with analytically calculated temperature.

The new model will be used to support design and performance evaluation of dual-cooled annular fuel.

REFERENCES

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